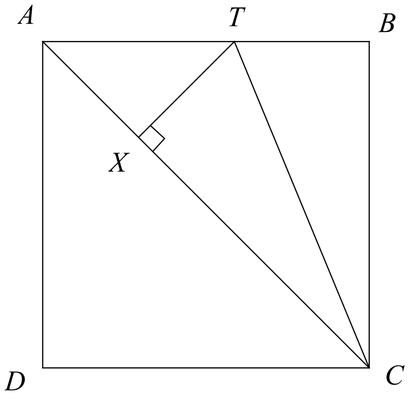
2005 YEAR 9 YEARLY

SECTION A (16 Marks) **Marks**

1 Find k if lines $2x - 5y + 1 = 0$ and $kx + 2y - 3 = 0$ are parallel. **3**

2 Given that $\dfrac{3}{3+\sqrt{3}} = p + q\sqrt{3}$, where p and q are rational, find the values of p and q. **3**

3. In the square $ABCD$ given, TC bisects $\angle ACB$ and TX is perpendicular to AC. Copy the diagram onto your answer sheet and prove that $AX = TB$. **5**



4. (a) Show that $(x + 2)$ is a factor of $P(x) = x^3 - 3x + 2$. **1**

   (b) Find the other factors of P(x) and hence sketch $P(x) = x^3 - 3x + 2$. **2**

   (c) Hence, or otherwise, solve $x^3 \leq 3x - 2$. **2**

SECTION B (16 Marks)

1. Fully factor $3x^4 - 48y^4$ **3**

2. Find the probability that two people were born on Friday if it is known that at least one was born on a Friday. **2**

3. Simplify $9x^{-2} \div 27x^{-3}$. **2**

4. Solve $x^2 - 6x + 4 = 0$ using the method of completing the square. **3**

5. A rhombus $ABCD$ has a perimeter of 32 cm. and diagonal AC of length 8 cm. Find the exact length of diagonal BD giving all reasons. **4**

6. Solve $9^{\frac{1}{x}} = 3\sqrt{3}$. **2**

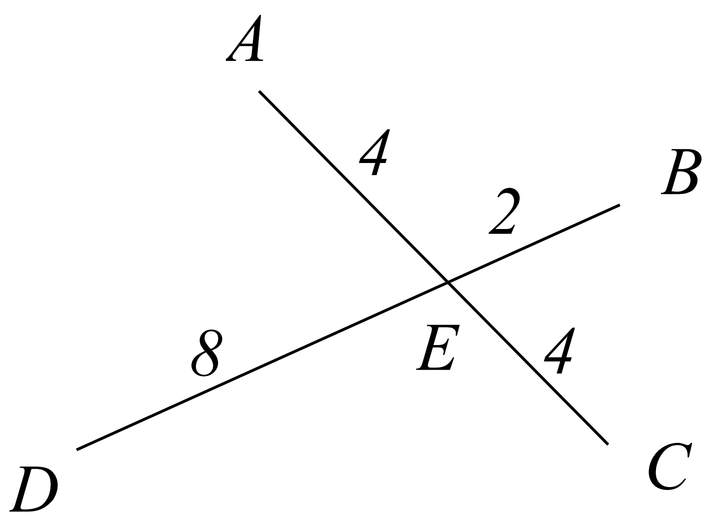
SECTION C (16 Marks)

1. Find the maximum value of $8-2x-x^2$ for all real values of x. **3**

2. A number is randomly selected from the first twenty positive integers. Find the probability that it is divisible by 2,3 or 4. **2**

3 (a) From the diagram given , show that $\Delta AED \parallel\!| \Delta BEC$. **3**

(b) Briefly explain why the points A,B,C and D lie on the circumference of a circle. **2**



4. The quadratic equation $x^2-4x+6=0$ has two solutions in the form of complex numbers $x=a+ib$ and $x=a-ib$, where $i=\sqrt{-1}$. Find the value of a^2+b^2. **3**

5. If $P=3^{2005}+3^{-2005}$ and $Q=3^{2005}-3^{-2005}$, evaluate P^2-Q^2. **3**

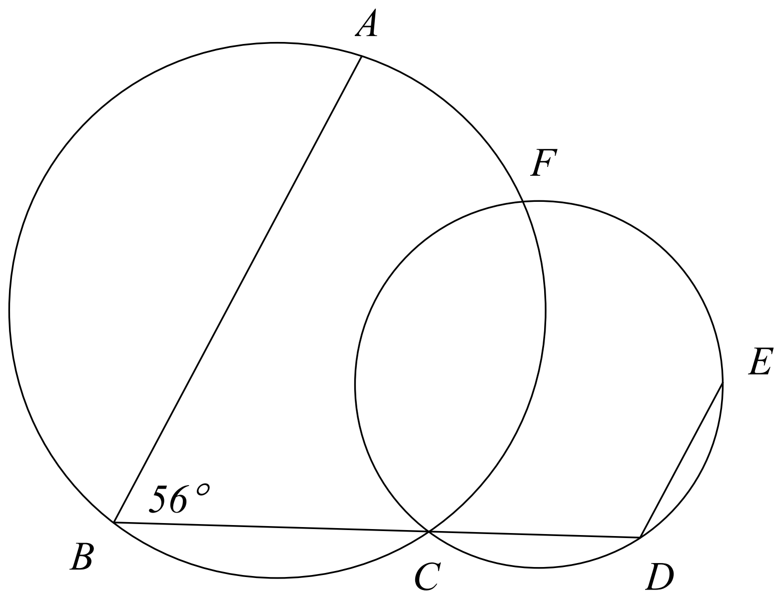
SECTION D (16 Marks)

1. Find the equation of the line passing through the point $(1,4)$ and perpendicular to the line $2x-3y-6=0$. **2**

2. Find all real solutions to the equation $4^x-5(2^x)-24=0$. **3**

3. The polynomial $P(x)=2x^3+ax^2+bx+1$ has a factor $(x+1)$ and a remainder of -5 when divided by $(x-2)$. Find a and b. **4**

4. In the diagram shown, $AB \parallel DE$ and $\angle ABC=56°$.



(a) Copy the diagram onto your answer sheet and find $\angle AFC$ giving reasons. **2**

(b) Prove that AFE is a straight line. **2**

5. If four horses eat four bales of hay in four days, how many days will it take for twenty horses to eat thirty bales of hay. **3**

SECTION E (16 Marks)

1. A triangle has a base length of $(x-1)$ cm and a height of $(2x-5)$ cm. Find its height if it has an area of $7\ \text{cm}^2$. **3**

2. **The answer to this question is to be done on the graph paper provided.**

   A factory makes two types of desk lamps; a standard model *S* and a deluxe model *D*. The factory can make up to 30 desk lamps per day. The standard model requires 2 minutes and the deluxe model 5 minutes to make the lamps on this machine, which is only available for 90 minutes per day for production.
   The profit on the standard model is $4 and on the deluxe model is $8.
   Let x be the number of model *S* and y be the number of model *D* made per day.

   (a) Given that $x \geq 0$ and $y \geq 0$, find equations for the constraints on time and on the number of lamps made. **2**

   (b) Find the equation of the profit line. **1**

   (c) Using a scale of 1cm = 5 units, graph the polygon using all of the constraints. **2**

   (d) How many of each model must be made to maximise the profit. **2**

3. Nine marbles numbered from 1 to 9 are placed in a bag and three are drawn out at random without replacement. Find the probability that the sum of the numbers on the marbles is odd. **3**

4. If $x > 0$, $y > 0$ and $x^2 - y^2 = 2xy$, find the exact value of $\frac{x}{y}$. **3**

END of PAPER

YEAR 9 YEARLY 2005

Section A

1. Let $m_1 = \frac{2}{5}$ and $m_2 = -\frac{k}{2}$.
If lines parallel, $m_1 = m_2$

$\therefore \frac{2}{5} = -\frac{k}{2}$

$\therefore k = -\frac{4}{5}$

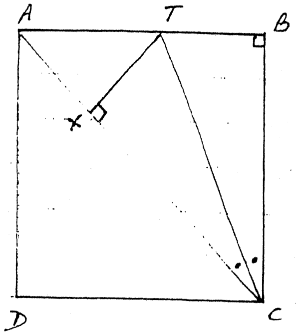
2. Now $\frac{3}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}} = p + q\sqrt{2}$

$\therefore \frac{3(3-\sqrt{3})}{6} = p + q\sqrt{2}$

$\therefore \frac{3}{2} - \frac{1}{2}\sqrt{3} = p + q\sqrt{2}$

$\therefore p = \frac{3}{2}$ and $q = -\frac{1}{2}$

3.



In $\triangle ATX$

$T\hat{A}X = 45°$ (diagonals of square bisect right angles)

$A\hat{X}T = 90°$ (adjacent angles on AXC supplementary)

$\therefore A\hat{T}X = 45°$ (angle sum of $\triangle ATX$)

$\therefore \triangle AXT$ is isosceles. (two equal angles)

$\therefore AX = XT$ (sides opposite equal angles in isosceles $\triangle AXT$)

In $\triangle TXC$, $\triangle TBC$

$T\hat{B}C = T\hat{X}C = 90°$ (given)

$X\hat{C}T = B\hat{C}T$ (TC bisects $A\hat{C}B$ given)

TC is common

$\therefore \triangle TXC \equiv \triangle TBC$ (AAS)

$\therefore TX = TB$ (corresponding sides of $\triangle TXC \equiv \triangle TBC$)

But $TX = AX$ (proven)

$\therefore AX = TB$

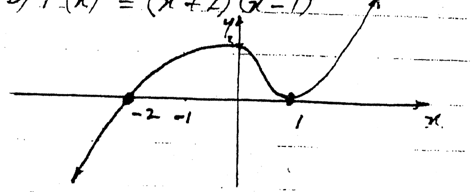
(a) If $(x+2)$ is a factor of $P(x)$, then $P(-2) = 0$

Now $P(-2) = -8 + 6 + 2$

$= 0$

$\therefore (x+2)$ is a factor of $P(x)$.

(b) $P(x) = (x+2)(x-1)^2$



(c) $x^3 \leq 3x - 2$ $\therefore x^3 - 3x + 2 \leq 0$

$\therefore x \leq -2$ or $x = 1$ from graph.

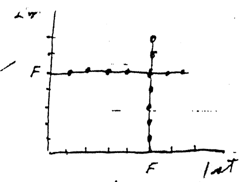
1. $3x^4 - 48y^4$

$= 3[x^4 - (2y)^4]$

$= 3(x^2 - 4y^2)(x^2 + 4y^2)$

$= 3(x-2y)(x+2y)(x^2+4y^2)$

2.



$P = \frac{1}{13}$

3. $9x^{-2} \div 27x^{-3}$

$= \frac{9}{x^2} \div \frac{27}{x^3}$

$= \frac{x}{3}$

4. $x^2 - 6x + 4 = 0$

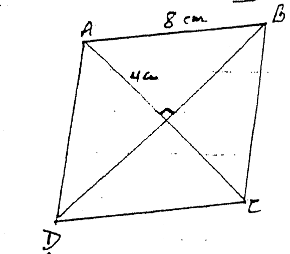
$\therefore x^2 - 6x + 9 = -4 + 9$

$\therefore (x-3)^2 = 5$

$\therefore x - 3 = \pm\sqrt{5}$

$\therefore x = 3 \pm \sqrt{5}$

5.



Now $AB = 8$ cm. (sides of rhombus equal, and perimeter is 32 cm. given)

Since diagonals of rhombus bisect each other at right angles

$\therefore \left(\frac{BD}{2}\right)^2 + 4^2 = 8^2$ (Pythagoras')

$\therefore BD^2 = 64 \times 3$

$\therefore BD = 8\sqrt{3}$ cm.

6. $9^{\frac{1}{x}} = 3\sqrt{3}$

$\therefore 3^{\frac{2}{x}} = 3^{\frac{3}{2}}$

$\therefore \frac{2}{x} = \frac{3}{2}$

$\therefore x = \frac{4}{3}$

Section C

1. $8 - 2x - x^2$

$= -(x^2 + 2x - 8)$

$= -[(x+1)^2 - 9]$

$= -(x+1)^2 + 9$

$\therefore$ MAXIMUM value of 9 (when $x = -1$).

OR

using $x = \frac{-b}{2a}$

$x = -1$

$\therefore 8 - 2x - x^2 = 8 + 2 - 1$

$= 9$

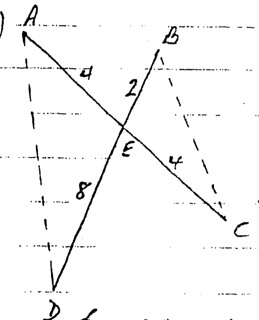
$\therefore$ maximum value of 9 (when $x = -1$)

2. Now 2, 4, ..., 20 are both divisible by 2 and 4, $\therefore \frac{10}{20}$.
Also 3, 6, 9, 12, 15, 18 divisible by 3
Since 6, 12, 18 are divisible by 2

$\therefore P = \frac{10}{20} + \frac{3}{20}$

$\therefore P = \frac{13}{20}$

3. (a)



In $\triangle AED$, $\triangle BEC$

$A\hat{E}D = B\hat{E}C$ (vertically opposite angles equal)

$\frac{AE}{ED} = \frac{4}{8} = \frac{1}{2}$ and $\frac{BE}{EC} = \frac{2}{4} = \frac{1}{2}$

$\therefore \frac{AE}{ED} = \frac{BE}{EC}$

$\therefore \triangle AED ||| \triangle BEC$ (sides about equal angles in same ratio)

(b) Since $\hat{A} = \hat{B}$ and angles at circumference of a circle in same segment are equal, $\therefore$ A, B, C and D are concyclic.

4 $x^2 - 4x + 6 = 0$

$\therefore x = \dfrac{4 \pm \sqrt{16-24}}{2}$

$= \dfrac{4 \pm \sqrt{-8}}{2}$

$= \dfrac{4 \pm 2\sqrt{2}\,i}{2}$

$\therefore x = 2 \pm i\sqrt{2}$

Since $x = a + ib$

$\therefore a = 2,\ b = \sqrt{2}$ and

$\underline{a^2 + b^2 = 6}$

5 $P^2 - Q^2 = (P-Q)(P+Q)$

$= \left[(3^{2005} + 3^{-2005}) - (3^{2005} - 3^{-2005})\right]\left[(3^{2005} + 3^{-2005}) + (3^{2005} - 3^{-2005})\right]$

$= (3^{2005} + 3^{-2005} - 3^{2005} + 3^{-2005})(3^{2005} + 3^{-2005} + 3^{2005} - 3^{-2005})$

$= (2 \times 3^{-2005})(2 \times 3^{2005})$

$= 4 \times 3^0$

$= \underline{4}$

Section D

1 Let equation be

$y - 4 = m_2(x-1)$

Gradient of $2x - 3y - 6 = 0$ is $m_1 = \frac{2}{3}$ $\therefore m_2 = -\frac{3}{2}$

$\therefore y - 4 = -\frac{3}{2}(x-1)$

$\therefore 2y - 8 = -3x + 3$

$\therefore \underline{3x + 2y - 11 = 0}$

2 $4^x - 5(2^x) - 24 = 0$

$\therefore (2^x)^2 - 5(2^x) - 24 = 0$

$\therefore (2^x - 8)(2^x + 3) = 0$

Since no real solution to $2^x + 3 = 0$

$\therefore 2^x - 8 = 0$

$\therefore 2^x = 2^3$

$\therefore \underline{x = 3}$

3 Since $(x+1)$ is a factor of $P(x)$ $\therefore P(-1) = 0$

$\therefore -2 + a - b + 1 = 0$

$\therefore a - b = 1$ — (1)

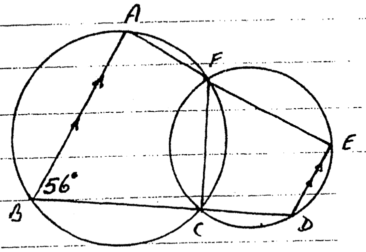
Since $P(2) = -5$

$\therefore 16 + 4a + 2b + 1 = -5$

$\therefore 2a + b = -11$ — (2)

$\therefore \underline{a = -\frac{10}{3},\ b = -\frac{13}{3}}$

4



(a) Join AF and FC.

$A\hat{F}C + 56° = 180°$ (opposite angles of cyclic quad ABCF supplementary)

$\therefore A\hat{F}C = 124°$

(b) Join EF.

$\therefore C\hat{D}E + 56° = 180°$ (co-interior angles are supplementary as AB∥DE)

$\therefore C\hat{D}E = 124°$

$\therefore C\hat{F}E = 56°$ (Opposite angles of cyclic quad CDEF supplementary)

Now $A\hat{F}C + C\hat{F}E = 180°$

Since adjacent angles are supplementary, $\therefore AFE$ is a straight line, ie; A, F and E are collinear.

5 4 horses eat 4 bales in 4 days,

$\therefore$ 4 horses eat 1 bale in 1 day,

$\therefore$ 1 horse eats $\frac{1}{4}$ bale in 1 day,

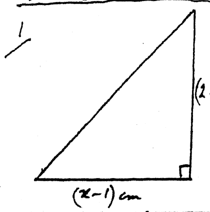
$\therefore$ 20 horses eat $\frac{20}{4}$ bales in 1 day,

$\therefore$ 20 horses eat 30 bales in 6 days.

$\therefore$ <u>6 days</u>

Section E

1



$\frac{1}{2}(x-1)(2x-5) = 7$

$\therefore (x-1)(2x-5) = 14$

$\therefore 2x^2 - 7x - 9 = 0$

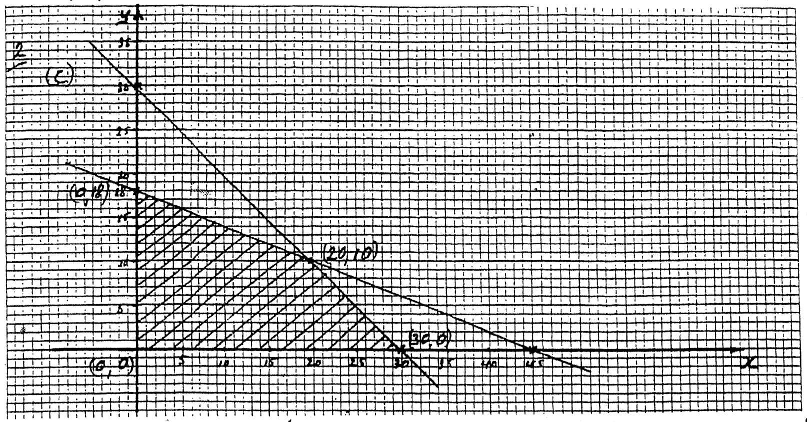
$\therefore (2x-9)(x+1) = 0$

$\therefore x = \frac{9}{2}$ or $x = -1$

Since sides must be a positive length, $\therefore x = \frac{9}{2}$ only.

$\therefore$ height is $(2 \times \frac{9}{2} - 5) = \underline{4\text{ cm}}$.

2



(a) Constraints on time and number of lamps:

$x + y \leq 30$ — (i)

$2x + 5y \leq 90$ — (ii)

(b) Equation of profit line is $P = 4x + 8y$

Solving (i) and (ii):

$y = 30 - x$

$2x + 5(30 - x) = 90$

$\therefore 3x = 60$

$\therefore x = 20$

$y = 10$

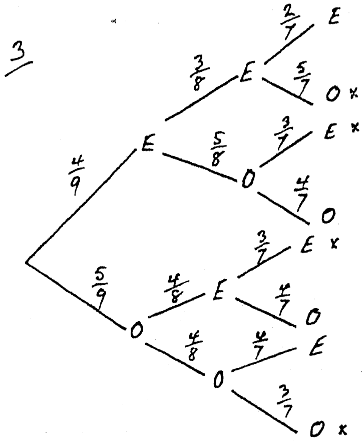
Testing for P at vertices:

(0,18) P = 144, (20,10) P = 160

(30,0) P = 120

$\therefore$ Maximum profit is P = ₵160 making 20 standard and 10 deluxe models.

3



Let the odd number on a marble be O and an even number be E. From the diagram:

$P = P(E,E,O) + P(E,O,E) + P(O,E,E) + P(O,O,O)$

$= 4\left(\frac{4}{9} \times \frac{3}{8} \times \frac{5}{7}\right)$

$\therefore P = \underline{\frac{10}{21}}$

4 Since $x^2 - y^2 = 2xy$

$\therefore \frac{x^2}{y^2} - 1 = \frac{2x}{y}$

$\therefore \frac{x^2}{y^2} - 2\left(\frac{x}{y}\right) - 1 = 0$

$\therefore \frac{x}{y} = \frac{2 \pm \sqrt{4+4}}{2}$

$\therefore \frac{x}{y} = 1 \pm \sqrt{2}$

Since $x > 0,\ y > 0$ $\therefore \frac{x}{y} = 1 + \sqrt{2}$