Section A (14 Marks) Begin a Separate sheet of paper

Marks

1. Expand and simplify $\left(2-3\sqrt{5}\right)^2$. **1**

2. Solve for x: $x^2-6x=0$. **1**

3. Simplify $\dfrac{8a^3-27}{3-2a}$. **2**

4. Express $\left(x^{-1}+y^{-1}\right)^{-1}$ as a rational expression with no negative indices. **2**

5. Make x the subject in $y=\dfrac{2x}{x-1}$. **2**

6. Find k if the point $(3,-4)$ lies on the line whose equation is $5x-4ky=12$. **2**

7. Solve for x : $\sqrt{2x+1}=\dfrac{7}{2}$. **2**

8. Factorise: $4a^2-b^2-4a+2b$. **2**

Section B (14 Marks) Begin a Separate sheet of paper

Marks

1. Find the angle X, to the nearest degree. **2**

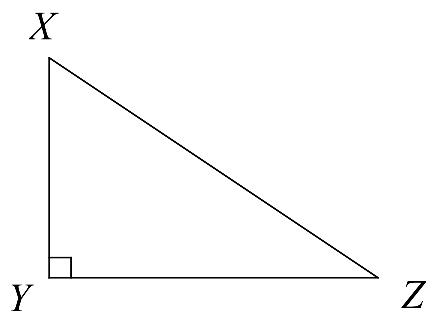


Diagram not

2. Find the length AC, to 3 significant figures. **2**

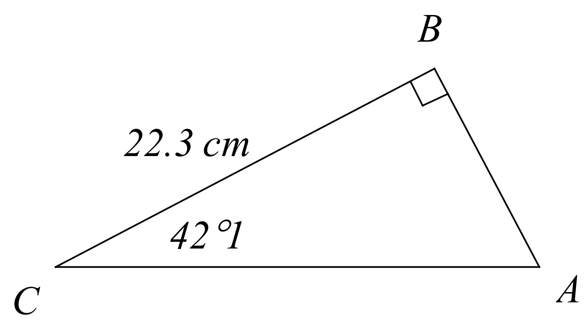


Diagram not

3. A ship sailing on a course bearing 142° is 5500 metres due north of a lighthouse (L). The ship continues on this course. **3**

What is the closest distance the ship will come to the lighthouse?
(Give answer to the nearest metre).

<u>First draw a diagram of the above information before answering the question.</u>

Marks

4. $(x+2)$ is a factor of the polynomial $P(x) = x^3 + ax + a^2$.

(i) Show that $a^2 - 2a - 8 = 0$. **1**

(ii) Hence, find all possible values of a. **1**

5. *ABCD* is a trapezium. *P* and *Q* are the midpoints of *AD* and *BD* respectively.
The line *PQ* is produced to *R*.

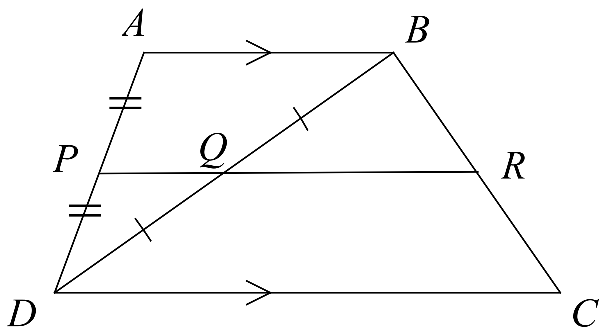


Diagram not

(i) Prove $PQ \parallel AB$ (giving reasons). **1**

(ii) Prove QR bisects BC (giving reasons). **2**

6. *ABCD* is a parallelogram and $BY=DX$.

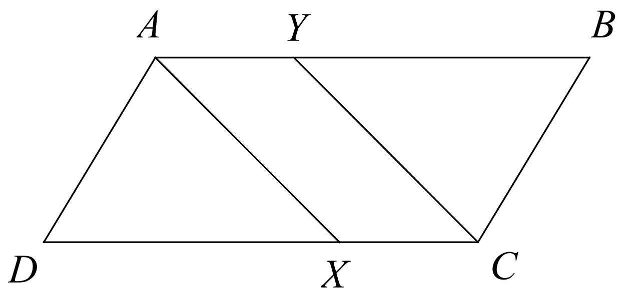


Diagram not

Prove *AYCX* is a parallelogram. **2**

<u>End of Section B. Go onto Section C</u>

<u>Section C</u> (14 Marks) Begin a Separate sheet of paper

Marks

1. The line L, whose equation is $3x+2y=6$, cuts the y-axis at $A(0, 3)$, as shown on the number plane. O is the origin.

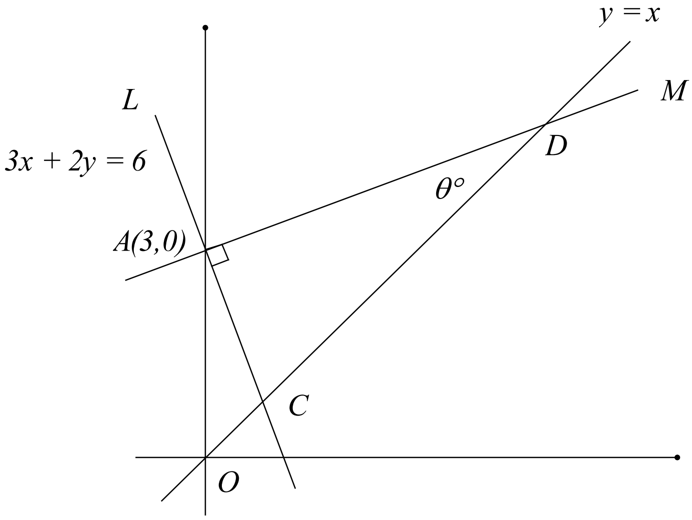


Diagram not

(i) The line M, through the point A is perpendicular to L. **2**
Find the equation of the line M.

(ii) The line $y=x$ cuts L and M at C and D respectively.
Find the coordinates of C and D. **2**

(iii) Find the exact length of CD. **2**

(iv) Find the area of the ΔAOD **1**

(v) If $\angle ADC = \theta^o$, find θ to the nearest degree and minute. **2**

2. The points P, Q, R, S and T lie on a circle. ST bisects $\angle RTV$. Let $\angle VTS = x^o$.

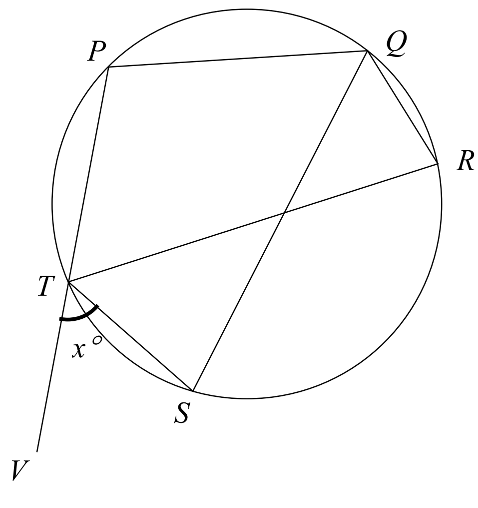


Diagram not
to scale

3

Copy the diagram onto your answer sheet and prove SQ bisects $\angle PQR$.

3. Solve for x: $\dfrac{2}{x-1}-\dfrac{4}{x^2-1}=1$ **2**

<u>Section D</u> (15 Marks) Begin a Separate sheet of paper

Marks

1. An enclosure is to be built adjoining a barn, as shown below. The walls of the barn meet at 135° and cannot be moved. The farmer has 117 metres of fencing available to build the enclosure so that $x + y = 117$, where x and y are as shown in the diagram.

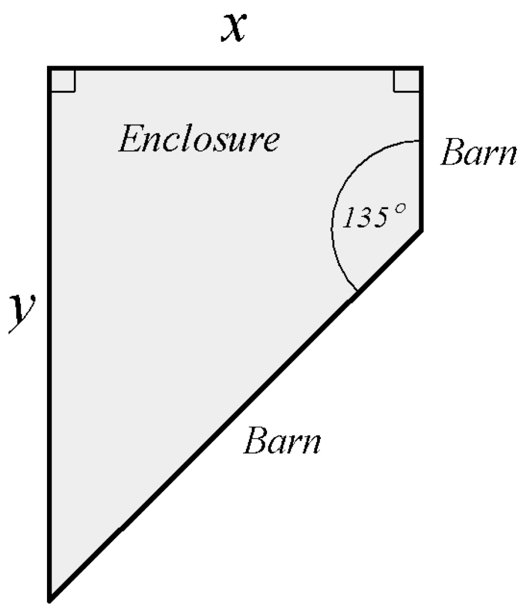


Diagram not to scale

(i) Show that the shaded area of the enclosure in square metres is given by: **2**

$$A = 117x - \frac{3}{2}x^2.$$

(ii) Show that the largest enclosured area which can be built occurs when y is twice the value of x. **2**

(iii) Find the area of the largest enclosure possible. **1**

2. A bag contains 2 red, 1 black and 1 white ball. Andrew selects one ball from the bag. He then selects a second ball but does not replace the first ball before the second ball is chosen.

(i) Draw a dot diagram, or otherwise, displaying all the possible outcomes. **1**

(ii) Find the probability that:

(a) both the selected balls are red. **1**

(b) at least one selected ball is red. **1**

(iii) What is the probability that Andrew had two red balls, if it is known that at least one ball was red? **2**

3. Use the remainder theorem to find the remainder when $x^3 + 2x^2 - 7$ is divided by $2x + 3$. **1**

4. (i) Sketch the graph of the polynomial $y = x^3 - 6x^2 + 9x - 4$. **3**

(ii) Hence, or otherwise, solve $x^3 - 6x^2 + 9x - 4 \geq 0$. **1**

<u>Section E</u> (15 Marks) Begin a Separate sheet of paper

Marks

1. In the diagram below, ΔABC is isosceles, $AB = AC = 1$, where $\angle BAC = 108^{\circ}$. The point D is chosen on BC such that $CD = 1$. Let $BC = x$ and $\angle ABC = \theta^{\circ}$.

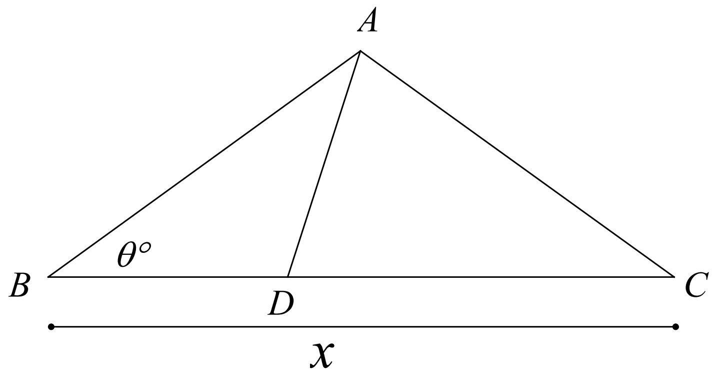


Diagram not

(i) Show that $\angle ADC = 72^{\circ}$, giving reasons. **2**

(ii) Show that $\Delta DBA \ ||| \ \Delta ABC$, giving reasons. **2**

(iii) Deduce from part (ii) that $x^2 - x - 1 = 0$. **1**

(iv) Find the value of x. **2**

2. A farmer prepares a rectangular garden which has length 30 metres and breadth 20 metres. Only celery and tomato plants are to be planted in the garden. The farmer intends to give each celery plant 0.3 m^2 of garden and each tomato plant 0.2m^2 of garden. No more than 2100 tomato plants may be used and the sum of the plants must not exceed 2500.

Let x represent the number of celery plants and y the number of tomato plants.

(i) Use the information above to complete the constraints. Copy these onto your answer sheet. **2**

$y \geq$ _________ .
$x \geq$ __________ .
$y \leq$ __________.
$x + y \leq$ _______.
and $3x + 2y \leq 6000$.

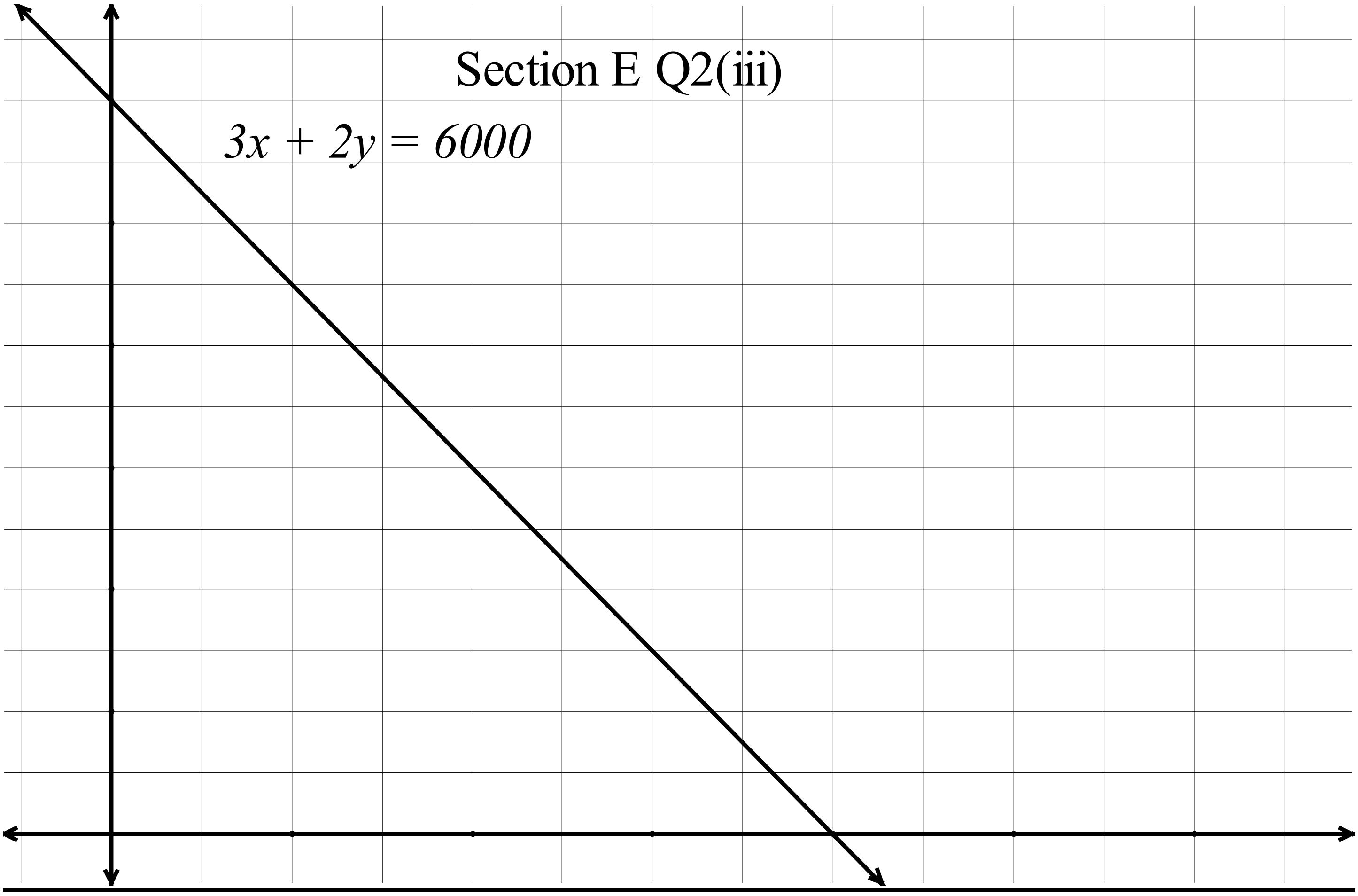
(ii) Explain why one of the above constraints is $3x + 2y \leq 6000$. **1**

(iii) **<u>Detach the graph paper at the end of the exam paper to answer this question.</u>** **2**
Graph the region which shows the number of celery and tomato plants that can be planted.

(iv) The farmer makes a profit of 96 cents on each celery plant and 72 cents on each tomato plant.
Write down an expression for profit (P) in terms of x and y. **1**

(v) Find his total maximum profit **and** find the number of each type of plant needed in the garden to produce his maximum profit. **2**

<u>END OF EXAM PAPER</u>



Section E Q2(iii)
$3x + 2y = 6000$

YR 9 - YEARLY EXAM

SOLUTIONS TO 2007 MATHS PAPER

Section (A) - 14 Marks

1. $(2-3\sqrt{5})^2 = 4 + (9\times 5) - 4\times 3\sqrt{5}$
$= 49 - 12\sqrt{5}$ $\boxed{1}$

2. $x(x-6) = 0$
$x = 0$ or $x = 6$ $\boxed{1}$

3. $\dfrac{(2a-3)(4a^2+6a+9)}{-(2a-3)}$ 1
$= -(4a^2+6a+9)$ 1 $\boxed{2}$

4. $\left(\frac{1}{x}+\frac{1}{y}\right)^{-1} = \left(\frac{y+x}{xy}\right)^{-1}$ 1
$= \dfrac{xy}{x+y}$ 1 $\boxed{2}$

5. $y = \dfrac{2x}{x-1}$
$yx - y = 2x$ 1
$yx - 2x = y$
$x(y-2) = y$
$x = \dfrac{y}{y-2}$ 1 $\boxed{2}$

6. $15 + 16k = 12$ 1
$16k = -3$
$k = \dfrac{-3}{16}$ 1 $\boxed{2}$

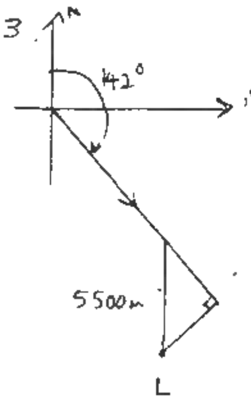
7. $\sqrt{2x+1} = \dfrac{7}{2}$
$2x+1 = \dfrac{49}{4}$ 1
$2x = \dfrac{45}{4}$
$x = \dfrac{45}{8}$ or $5\frac{5}{8}$ 1 $\boxed{2}$

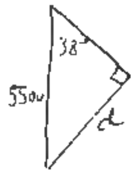
8. $(2a-b)(2a+b) - 2(2a-b)$ 1
$= (2a-b)(2a+b-2)$ 1 $\boxed{2}$

Section (B) - 14 Marks

1. $\tan x = \dfrac{16}{71}$ 1
$x = 13^\circ$ (nearest degree) 1 $\boxed{2}$

2. $\cos 42^\circ 17' = \dfrac{22\cdot 3}{AC}$ 1
$AC = \dfrac{22\cdot 3}{\cos 42^\circ 17'}$
$AC = 30.1$ (3sf) 1 $\boxed{2}$

3. 

Now in triangle 

$\sin 38^\circ = \dfrac{d}{5500}$ 1
$\therefore d = 5500 \times \sin 38^\circ$
$\therefore d = 3386\text{ m}$ (nearest metre) 1 $\boxed{3}$

4. (i) If $(x+2)$ is a factor then $P(-2) = 0$
ie $(-2)^3 - 2a + a^2 = 0$ 1 $\boxed{1}$
$a^3 - 2a + 8 = 0$

(ii) $(a-4)(a+2) = 0$
$a = 4$ or -2 1 $\boxed{1}$

5. (i) $PQ \parallel AB$ (a line joining the midpoints of 2 sides of a triangle is parallel to the third side.) 1 $\boxed{1}$

Section (B) Continued

5 (ii) $PQ \parallel AB \parallel CD$ (1 pair of opposite sides of trapezium)

$CD \parallel PR$ (PQ produced)

1 $\therefore \frac{DQ}{QB} = \frac{CR}{RB}$ (a line parallel to one side of a triangle divides other 2 sides in same ratio)

1 But $\frac{DQ}{QB} = \frac{1}{1}$ (Q is midpoint BD)

$\therefore \frac{CR}{RB} = \frac{1}{1}$ also.

$\therefore$ R is the midpoint of BC

[2] so, QR bisects BC

6 $AB = CD$ (opposite sides of parm. ABCD are equal)

1 $AY = XC$ (subtraction of equal lengths BY & DX from equal sides AB & CD)

$AY \parallel XC$ (opposite sides of parm ABCD are equal)

1 $\therefore$ AYCX is a parm (One pair of opposite sides are equal & parallel)

[2]

Section (C) (14 marks)

1 (i) gradient of L is $-\frac{3}{2}$ 1

$\therefore$ gradient of M is $\frac{2}{3}$.

$\therefore$ Eq^n of M is:

$y = mx + b$

ie $y = \frac{2}{3}x + 3$ 1 [2]

(ii) L: $3x + 2y = 6$ M: $y = \frac{2}{3}x + 3$

$5x = 6$ $x = \frac{2}{3}x + 3$

$x = \frac{6}{5}$ $\frac{1}{3}x = 3$

$x = 9$

$\therefore C\left(\frac{6}{5}, \frac{6}{5}\right)$ $D(9, 9)$

1 1

[2]

(iii) $CD = \sqrt{\left(9 - \frac{6}{5}\right)^2 + \left(9 - \frac{6}{5}\right)^2}$ 1

$= \sqrt{2 \times 60\frac{21}{25}}$

$= \sqrt{\frac{3042}{25}} = \frac{\sqrt{3042}}{5}$ 1

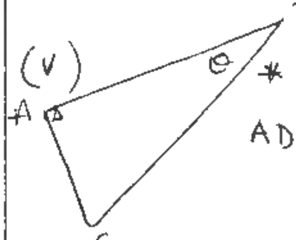
[2]

(iv) $A_{\Delta AOD} = \frac{1}{2} \times AO \times 9$

$= \frac{1}{2} \times 3 \times 9$

$= \frac{27}{2}\ u^2$

(v)



* Need AD or AC

$AD = \sqrt{9^2 + (9-3)^2}$

$= \sqrt{117}$ 1

$\therefore \cos\theta = \frac{\sqrt{117}}{\frac{\sqrt{3042}}{5}}$

$\theta = 11°19'$ 1 [2]

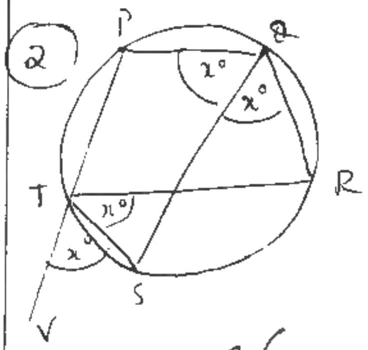
OR $AC = \sqrt{\left(\frac{6}{5} - 0\right)^2 + \left(\frac{6}{5} - 3\right)^2}$

$= \frac{\sqrt{117}}{5}$

$\therefore \sin\theta = \frac{\frac{\sqrt{117}}{5}}{\frac{\sqrt{3042}}{5}}$

$\theta = 11°19'$

2



$\angle RTS = x°$ (ST bisects $\angle RTV$)

$\angle SQR = x°$ (angles subtended to the circumference of a circle by the same segment are equal.) 1

$\angle PQS = x°$ (ext. angle of a cyclic quad PQST equals the interior opposite 1

(3) $\frac{2}{x-1} - \frac{4}{x^2-1} = 1$

1 $\quad 2(x+1) - 4 = x^2 - 1, \quad x \neq \pm 1$

$2x + 2 - 4 = x^2 - 1$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

½ $\quad x = 1$

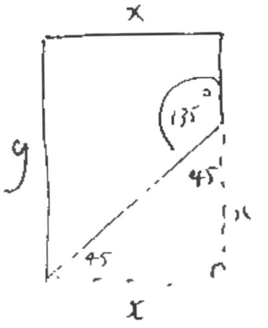
but $x \neq 1$,

½ $\quad$ so there is no solⁿ.

[2]

Section D (15 Marks)

(1)



(i)

1 $\quad A = xy - \frac{1}{2}x^2$

but $x + y = 117$

$y = 117 - x$

1 $\quad \therefore A = x(117 - x) - \frac{1}{2}x^2$

$= 117x - x^2 - \frac{1}{2}x^2$

[2] $\quad A = 117x - \frac{3}{2}x^2$

(ii) Since this is a quadratic function which is concave down then the maximum occurs at the vertex.

1 $\quad$ ie $x = \frac{-b}{2a} = \frac{-117}{2 \times -\frac{3}{2}} = 39$

$\therefore y = 117 - 39 = 78$

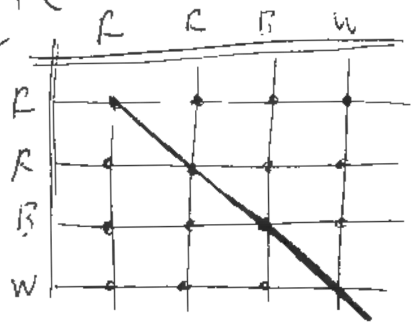
1 $\quad = 2 \times 39$

[2] $\quad \therefore$ y is twice the value of x.

(iii) Largest area $= 117 \times 39 - \frac{3}{2} \times 39^2$

[1] $\quad = 2281\frac{1}{2}\ m^2$

(2) (i)



[1]

(ii) a) $P(RR) = \frac{2}{12} = \frac{1}{6}$ [1]

b) $P(\text{at least 1 Red}) = \frac{10}{12} = \frac{5}{6}$ [1]

(iii) $P\left(\begin{array}{l}\text{2 Red, if known} \\ \text{at least 1 is Red}\end{array}\right) = \frac{2}{10} = \frac{1}{5}$ [2]

(3) Let $P(x) = x^3 + 2x^2 - 7$

$P(-\frac{3}{2}) = (-\frac{3}{2})^3 + 2(-\frac{3}{2})^2 - 7$

$= -5\frac{7}{8}$ [1]

$\therefore$ Remainder is $-5\frac{7}{8}$

(4) (i) Let $P(x) = x^3 - 6x^2 + 9x - 4$

$P(1) = 1 - 6 + 9 - 4 = 0$

$\therefore (x-1)$ is a factor

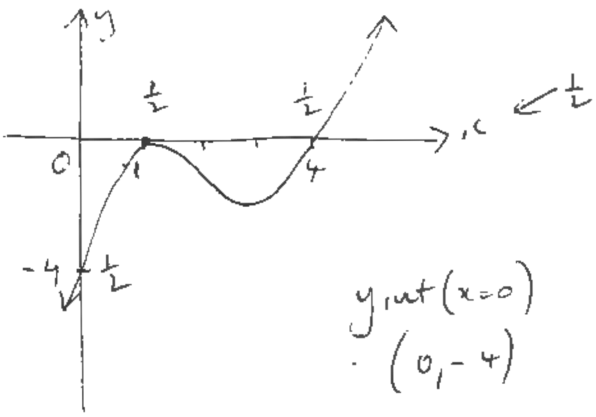
$$\begin{array}{r|l} & x^2 - 5x + 4 \\ \hline x - 1 & x^3 - 6x^2 + 9x - 4 \\ & x^3 - x^2 \\ \hline & -5x^2 + 9x - 4 \\ & -5x^2 + 5x \\ \hline & 4x - 4 \\ & 4x - 4 \\ \hline \end{array}$$

$\therefore P(x) = (x-1)(x^2 - 5x + 4)$

$= (x-1)(x-1)(x-4)$

$\therefore P(x) = (x-1)^2(x-4)$ $\quad$ 1

$y = (x-1)^2(x-4)$



y int ($x=0$)

$(0, -4)$

$\left.\begin{array}{l} \frac{1}{2} - (1,0) \\ \frac{1}{2} - (4,0) \\ \frac{1}{2} - (0,-4) \\ \frac{1}{2} - \text{labelling axes} \end{array}\right\}$ $\boxed{2}$

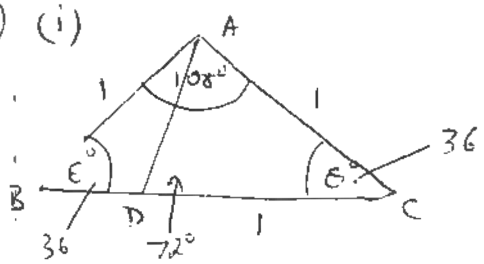
(ii) $x^3 - 6x^2 + 9x - 4 \geq 0$

Sol^n: $x \geq 4$ and $x = 1$ $\boxed{1}$

$\frac{1}{2}$ $\frac{1}{2}$

Section (E)

(1) (i)



$\angle ACD = \frac{180-108}{2}$ (angle sum of Δ is 180°; angles opposite equal sides of ΔABC are equal) 1

$\therefore \angle ACD = 36°$

$\angle ADC = \frac{180-36}{2}$ (angle sum of Δ is 180°, angles opposite equal sides of ΔACD are equal) 1

$\boxed{2}$ $\therefore \angle ADC = 72°$

(ii) In ΔABC & DBA:

$\frac{1}{2}$ $\angle B$ is common

$\frac{1}{2}$ $\angle ADB = 108°$ (angle sum of straight angle is 180°)

$\angle BAC = 108°$ (given)

$\frac{1}{2}$ $\therefore \angle ADB = \angle BAC$

$\frac{1}{2}$ $\therefore \Delta DBA \parallel\!\mid \Delta ABC$ (equiangular) $\boxed{2}$

(iii) $\frac{BC}{AB} = \frac{AC}{BD}$

$\boxed{1}$ $\frac{x}{1} = \frac{1}{x-1}$ (corresponding sides of similar Δs are in proportion)

$x(x-1) = 1$

$x^2 - x - 1 = 0$

(iv) $x = \frac{1 \pm \sqrt{1+4.1.1}}{2}$

$x = \frac{1 \pm \sqrt{5}}{2}$ 1

but $x > 0 \implies x = \frac{1+\sqrt{5}}{2}$ only. 1 $\boxed{2}$

(2) (i) $y \geq 0$ $\frac{1}{2}$

$x \geq 0$ $\frac{1}{2}$

$y \leq 2100$ $\frac{1}{2}$ $\boxed{2}$

$x + y \leq 2500$ $\frac{1}{2}$

(ii) Area $= 30 \times 20 = 600$

Area of celery $= 0.3x$

Area of tomatoes $= 0.2y$

$\therefore 0.3x + 0.2y = 600$ $\boxed{1}$

$3x + 2y = 6000$

(iii) See attached sheet [2]

(iv) $P = 0.96x + 0.72y$
(where P is in dollars) OR

$P = 96x + 72y$
(where P is in cents) [1]

(v) Max. profit occurs at the point (1000, 1500)
$P = 0.96 \times 1000 + 0.72 \times 1500$
$= \$2040$. 1

∴ No. of celery is 1000 and
No. of tomato is 1500 1

[2]

End of Exam paper

SECTION E Question 2 (iii) **Name:** ______________________

Class: ______________________

Remove this graph and attach to your solution for Section E Q2 (iii)