Year 9 Yearly Examination 2008

Section A (15 MARKS)

Marks

(a) Evaluate $\dfrac{5+\sqrt{3}}{6-\pi}$ giving your answer correct to 2 decimal places. 1

(b) Factorise $a^3 - 8b^3$. 1

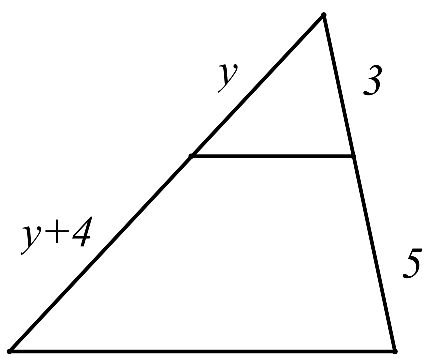
(c) Evaluate $\sin 32°44'$ giving your answer correct to 3 decimal places. 1

(d) Write $27^{-\frac{4}{3}}$ as a fraction in simplest form. 1

(e) Draw a neat sketch of a frequency polygon that has positive skew. 1

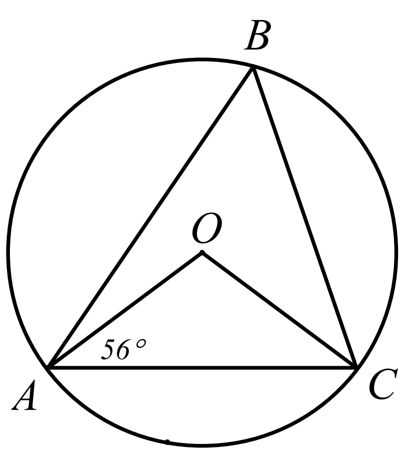
(f) Solve for p: $(3p-2)^2 = 5$. 2

(g) Find the value of y giving reasons. 2



(h) Solve for t: $t - 3 = \sqrt{4t-7}$. 3

(i) O is the centre of the circle and $\angle OAC = 56°$. 3
Copy the diagram onto your answer sheet and find the size of $\angle ABC$ giving reasons.



Section B (START A NEW PAGE) (15 MARKS)

Question 1 Marks

$OABC$ is a quadrilateral with verticies $O(0,0)$, $A(10,10)$, $B(8,24)$ and $C(-2,14)$.

(a) Prove that $OC \parallel AB$. 1

(b) Prove that $OC = AB$. 1

(c) Prove that $OB \perp AC$. 2

(d) What type of quadrilateral is $OABC$? (Give reason) 1

Question 2 Marks

On Sunday morning Jan rides 60km from Hornsby to Gosford at a constant speed. In the afternoon she returns from Gosford to Hornsby by the same route but at a speed that is 4km/hour slower with the result that the journey takes her 30 minutes longer.

(a) Let Jan's speed in the morning be v km/h. Write an expression involving v for the time, in hours, that Jan needs to complete the morning journey. 1

(b) Write a similar expression involving v for the time, in hours, that Jan needs to complete the afternoon journey. 1

(c) Write an equation and solve it to find Jan's speed for the morning journey. 3

Question 3 Marks

On a table are ten sealed envelopes. Five of the envelopes contain a \$10 note, three contain a \$20 note and the remaining two contain a \$50 note. A contestant selects two of the envelopes at random.

(a) Draw a probability tree diagram to illustrate the possible outcomes. 2

(b) Find the probability that the contestant wins \$100. 1

(c) Find the probability that the contestant wins less than \$40. 2

Section C (START A NEW PAGE) (15 Marks)

Question 1 Marks

(a) **<u>On the grid provided</u>**, sketch the region defined by the inequalities $2x + y \geq 120$, $y \geq x$ and $y \leq 2x$. 3

(b) For values of x and y defined by the above region find the minimum value of V when $V = 5x + 4y - 100$. 2

Question 2 Marks

A university student carrying out research on fish in a local steam caught, weighed and released 25 fish. Their weights, rounded off to the nearest gram, are recorded in the stem-and-leaf plot below.

Stem	Leaf
0	4 7 7 9
1	0 2 2 5 5 6
2	0 0 1 2 3 4 5 5 5 7
3	0 1 2 4
4	2

(a) Find the (i) range, (ii) median and (iii) interquartile range of the data. 3

(b) Display the above data in a box-and-whisker plot. 2

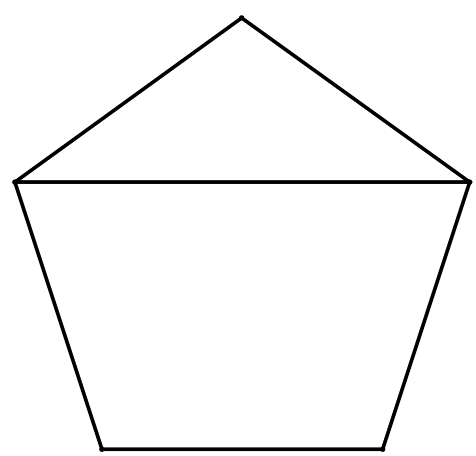
(c) With the aid of a calculator, find the standard deviation of the given data. Give your answer to one decimal place 1

Question 3 Marks

ABCDE is a regular pentagon.

(a) Copy the diagram onto your answer sheet and prove that $\angle ABC = 108°$. 1

(b) Prove that *AC* is parallel to *ED*. 4



Section D (START A NEW PAGE) (15 Marks)

Question 1 Marks

The polynomial $A(x) = px^3 + qx^2 - 2$ is divisible by $x - 1$ and leaves a remainder of 18 when divided by $x + 2$. Find the value of p and q. 4

Question 2 Marks

Express $\dfrac{3\sqrt{2}}{8 - 5\sqrt{2}}$ as a fraction in simplest form with a rational denominator. 3

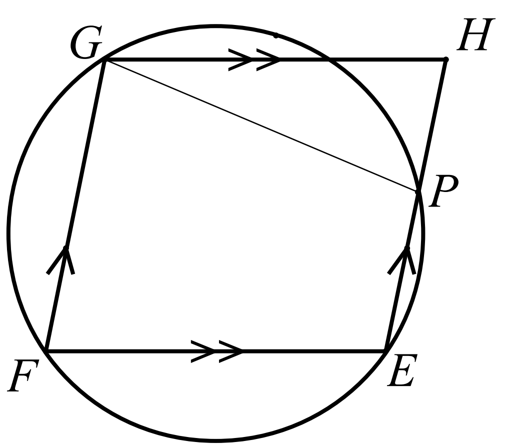
Question 3 Marks

(a) Sketch the curve $y = (x+2)^2(x-4)$, clearly showing all intercepts with the coordinate axes. 2

(b) Hence solve the inequality $(x+2)^2(x-4) \geq 0$. 2

Question 4 Marks

EFGH is a parallelogram. A circle passing through the vertices *E*, *F* and *G* cuts the side *EH* at *P*.



(a) Copy the diagram onto your answer sheet and prove that $\angle GHP = \angle GPH$. 2

(b) Hence prove that $EF = PG$. 2

Section E (START A NEW PAGE) (15 Marks)

Question 1 Marks

Fully factorise $9m^2 - 6m - 4n^2 + 4n$. 3

Question 2 Marks

The bearings and distances of two mountains viewed from my house are measured. Mount Visible is 6.4 km from my house and its bearing is $N25°E$, while Mount Mighty is 4.5 Km from my house and bears $N65°W$.

(a) Draw a neat diagram of the above information, clearly showing all angles and distances. 2

(b) Find, correct to the nearest 100m, the distance between Mount Visible and Mount Mighty. 2

(c) Find, correct to the nearest degree, the bearing of Mount Visible when viewed from Mount Mighty. 3

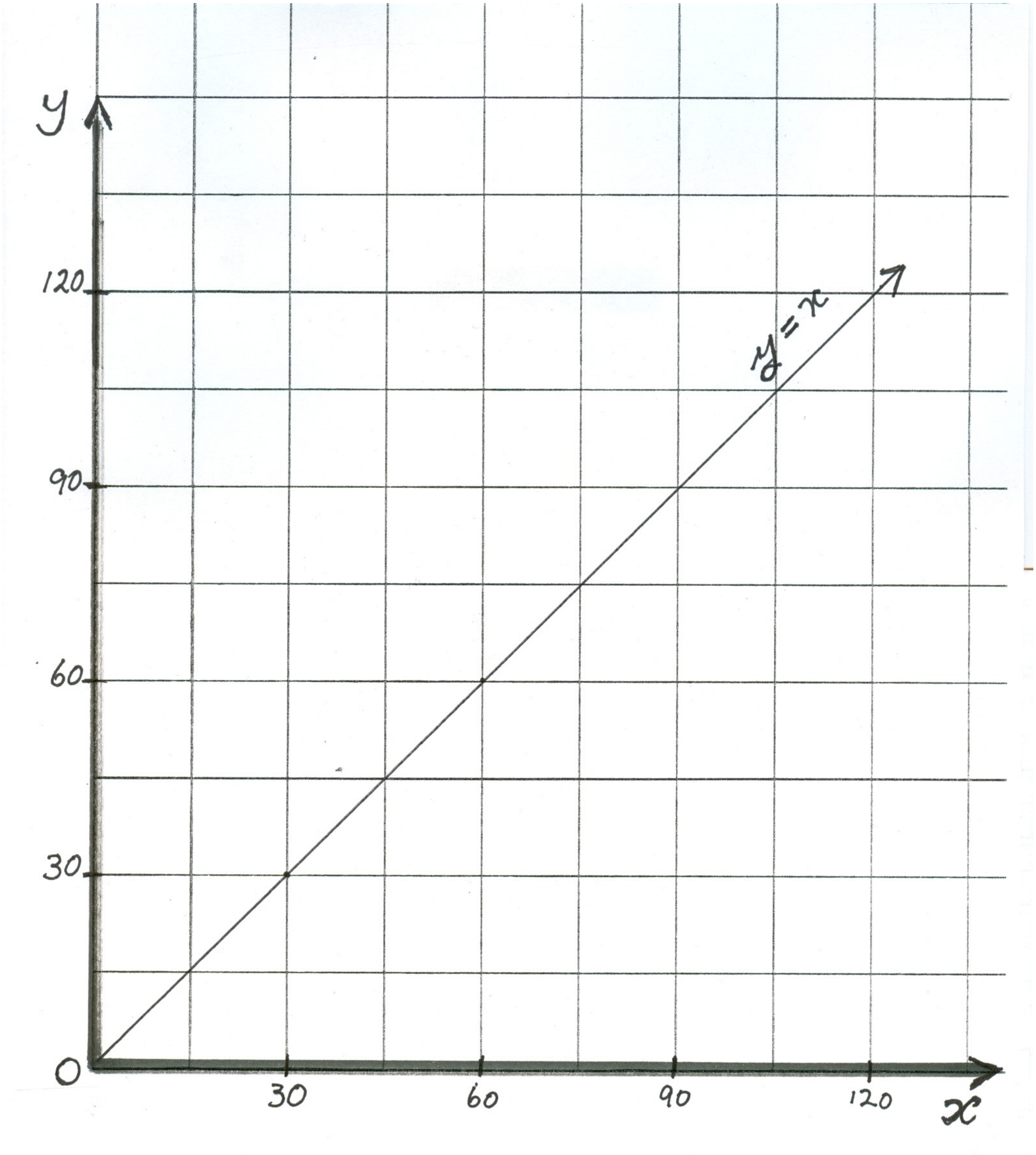
Question 3 Marks

(a) The line $3x + 4y - 48 = 0$ crosses the x-axis at point A and the y-axis at point B. Find the coordinates of points A and B. 1

(b) Point $P(r, s)$ lies on the interval AB. If the interval OP, where O is the origin, divides ΔAOB into two smaller triangles ΔAOP and ΔBOP whose areas are in the ratio 3:5, find the coordinates of point P. 4

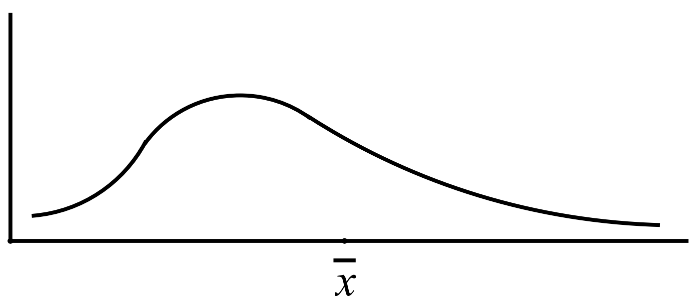
This is the end of the examination

SECTION C QUESTION 1(a)
Complete the answer to Section C Question 1(a) on this grid
Return this page with the rest of your answers to Section C



Year 9 Yearly Examination 2008 – Solutions

Section A (15 MARKS)

		Marks
(a)	2.36	1
(b)	$(a-2b)(a^2+2ab+4b^2)$	1
(c)	0.541	1
(d)	$\dfrac{1}{(\sqrt[3]{27})^4}=\dfrac{1}{81}$	1
(e)		1
(f)	$(3p-2)^2=5$ $3p-2=\pm\sqrt{5}$ $3p=2\pm\sqrt{5}$ $p=\dfrac{2\pm\sqrt{5}}{3}$	2
(g)	$\dfrac{y}{y+4}=\dfrac{3}{5}$ (interval parallel to side of triangle divides other sides in same ratio) $5y=3y+12$ $2y=12$ $y=6$ * Could prove that the triangles are similar then use ratios of corresponding sides	2

(h) $t-3=\sqrt{4t-7}$ 3

$(t-3)^2=4t-7$

$t^2-6t+9=4t-7$

$t^2-10t+16=0$

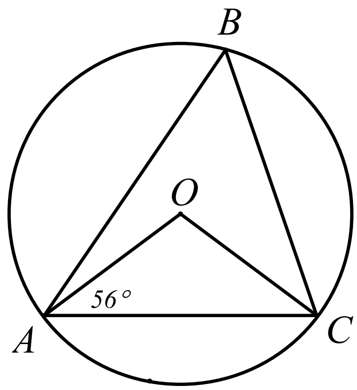
$(t-2)(t-8)=0$

$t=2 \text{ or } 8$

but $t-3\geq 0$, i.e. $t\geq 3$

$\therefore\ t=8$

(i) 3



$OA=OC$ (radii of circle)

$\angle OCA=56°$ (equal angles are opposite equal sides in ΔAOC)

$\angle AOC+112°=180°$ (angle sum of $\Delta AOC=180°$)

$\angle AOC=68°$

$\angle ABC=34°$ (angle at circumference is half angle at centre standing on same arc AC)

Section B (START A NEW PAGE) (15 MARKS)

Question 1 Marks

(a) 1

$$m(OC)=\frac{14-0}{-2-0}$$
$$=-7$$
$$m(AB)=\frac{24-10}{8-10}$$
$$=-7$$

$\therefore OC \parallel AB$ (equal slopes)

(b) 1

$$d(OC)=\sqrt{(-2-0)^2+(14-0)^2}$$
$$=\sqrt{200}$$
$$=10\sqrt{2}$$
$$d(AB)=\sqrt{(8-10)^2+(24-10)^2}$$
$$=\sqrt{200}$$
$$=10\sqrt{2}$$

$\therefore OC=AB$ (equal lengths)

(c) $m(OB)=\frac{24-0}{8-0}$ 2

$=3$

$m(AC)=\frac{14-10}{-2-10}$

$=-\frac{1}{3}$

$m(OB)\times m(AC)=3\times\left(-\frac{1}{3}\right)$

$=-1$

$\therefore OC\perp AB$ (product of slopes equals -1)

(d) Rhombus 1

$OC \parallel AB$ and $OC=AB$

$\therefore OABC$ is a parallelogram (pair of sides equal and parallel)

but $OC\perp AB$

$\therefore OABC$ is a rhombus (parallelogram with perpendicular diagonals)

Question 2 Marks

(a) $\frac{60}{v}$ 1

(b) $\frac{60}{v-4}$ 1

(c) $\frac{60}{v-4}-\frac{60}{v}=\frac{1}{2}$ 3

$120v-120(v-4)=v(v-4)$

$480=v^2-4v$

$v^2-4v-480=0$

$(v-24)(v+20)=0$

$v=-20$ or 24

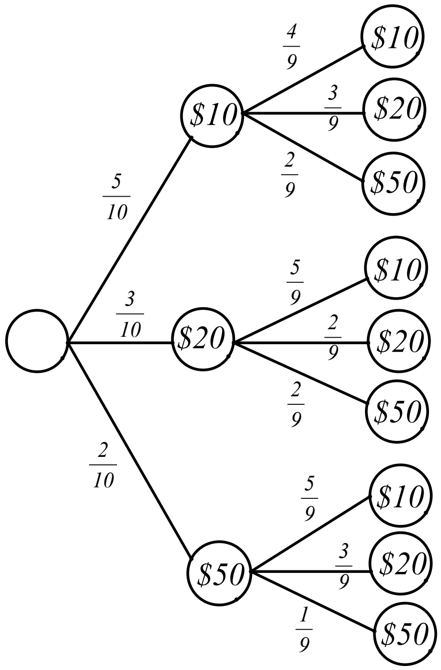
but $v>0$

$\therefore v=24$

$speed=24km/hr$

Question 3 Marks

(a) 2



(b) $P(\$100) = \frac{2}{10} \times \frac{1}{9}$ 1

$= \frac{1}{45}$

(c) $P(< \$40) = P(\$20) + P(\$30)$ 2

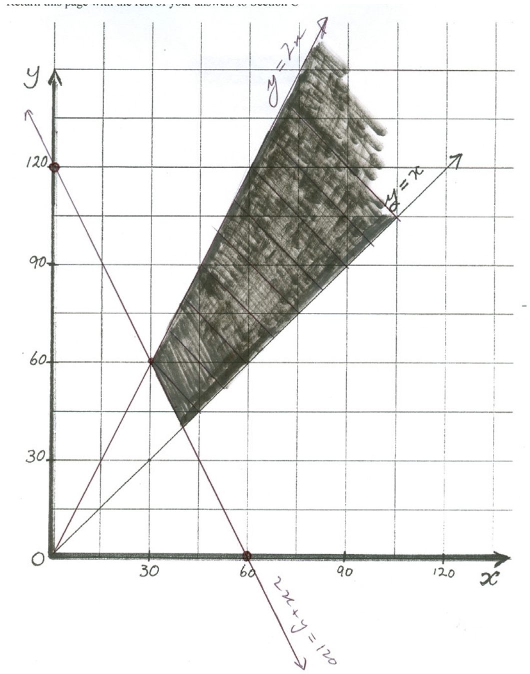
$= \frac{5}{10} \times \frac{4}{9} + \frac{5}{10} \times \frac{3}{9} + \frac{3}{10} \times \frac{5}{9}$

$= \frac{5}{9}$

Section C (START A NEW PAGE) (15 Marks)

Question 1 Marks

(a) 3



(b) At A 2

$2x + y = 120$ ……… (1)

$y = x$ ……… (2)

sub. (2) into (1)

$3x = 120$

$x = 40$

$y = 40$

vertex $(40,40)$

At B

$2x + y = 120$ ……… (1)

$y = 2x$ ……… (2)

sub. (2) into (1)

$4x = 120$

$x = 30$

$y = 60$

vertex $(30,60)$

x	y	$V = 5x + 4y - 100$
40	40	260
30	60	290

Minimum value of V =260

Question 2 Marks

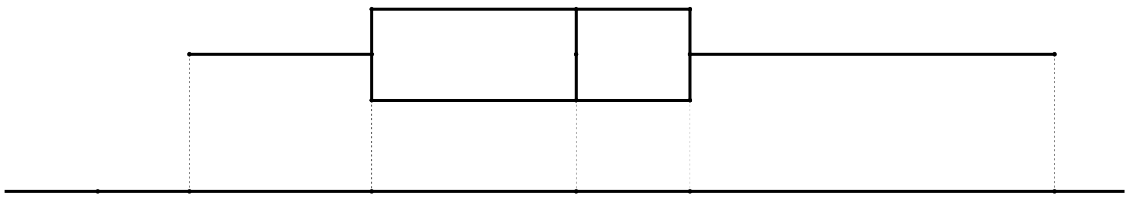
(a) (i) range = 38 3

(ii) median = 21

(iii) interquartile range of the data = 26 – 12

= 14

(b) 2



(c) $\sigma_n = 9.4 \quad (\text{to 1 decimal place})$ 1

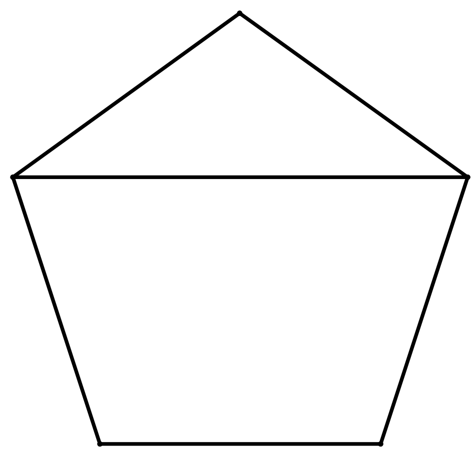
Question 3 Marks

(a) 1

$$\angle ABC = \frac{180(n-2)^\circ}{n}$$

$$= \frac{180(5-2)^\circ}{5}$$

$$= 108^\circ$$



(b) 4

$AB = BC \begin{pmatrix}\text{all sides of regular} \\ \text{pentagon are equal}\end{pmatrix}$

$\angle BAC = \angle BCA = x^\circ \begin{pmatrix}\text{equal angles are} \\ \text{opposite equal sides} \\ \text{of } \Delta ABC\end{pmatrix}$

$2x + 108 = 180 \ (\text{angle sum of } \Delta ABC = 180^\circ)$

$x = 36$

$\angle CAE + 36^\circ = 108^\circ \begin{pmatrix}\text{angles at vertices} \\ \text{equal } 108^\circ\end{pmatrix}$

$\angle CAE = 72^\circ$

$\angle CAE + \angle AED = 72^\circ + 108^\circ$

$= 180^\circ$

$AC \parallel ED \begin{pmatrix}\text{cointerior angles are} \\ \text{supplementary}\end{pmatrix}$

Section D (START A NEW PAGE) (15 Marks)

Question 1 Marks

4

$p(1)=0 \quad \Rightarrow \quad p+q-2=0$

$p+q=2 \quad \ldots\ldots \quad (1)$

$p(-2)=18 \quad \Rightarrow \quad -8p+4q-2=18$

$-2p+q=5 \quad \ldots\ldots \quad (2)$

$(1)-(2)$

$3p=-3$

$p=-1$

sub. into $(1) \quad \therefore \quad -1+q=2$

$q=3$

$\therefore p=-1$ and $q=3$

Question 2 Marks

3

$$\frac{3\sqrt{2}}{8-5\sqrt{2}}=\frac{3\sqrt{2}}{8-5\sqrt{2}}\times\frac{8+5\sqrt{2}}{8+5\sqrt{2}}$$

$$=\frac{3\sqrt{2}\left(8+5\sqrt{2}\right)}{8^2-\left(5\sqrt{2}\right)^2}$$

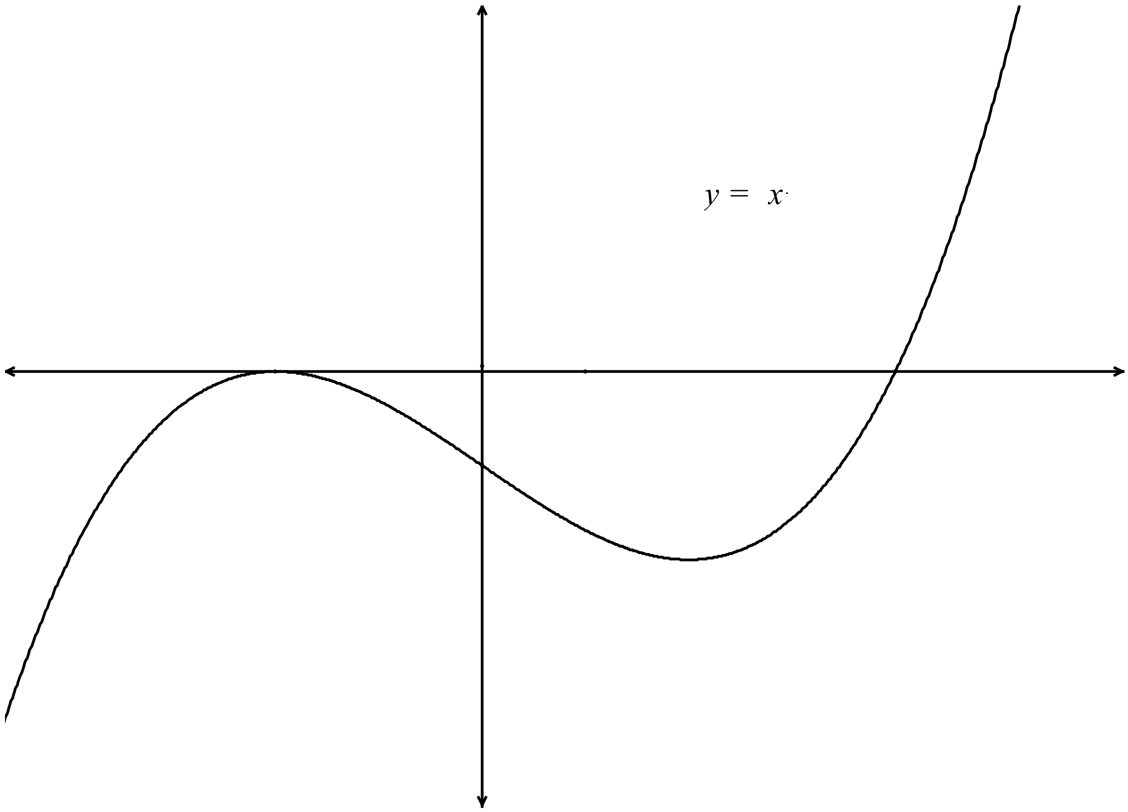
$$=\frac{24\sqrt{2}+30}{64-50}$$

$$=\frac{30+24\sqrt{2}}{14}$$

$$=\frac{15+12\sqrt{2}}{7}$$

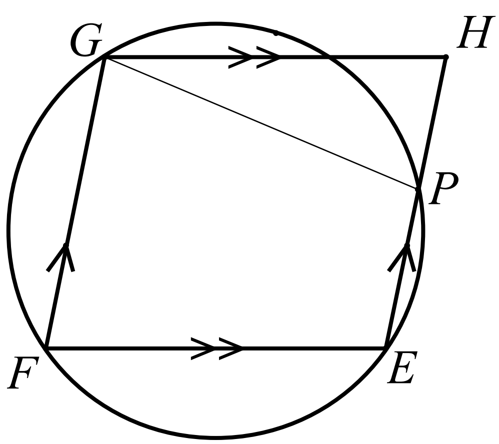
Question 3 Marks

(a) 2



(b) $x=-2$ or $x \geq 4$ 2

Question 4 Marks



(a) 2

$\angle GHP = \angle GFE$ (opposite angles of parallelogram $EFGH$ are equal)

$\angle GPH = \angle \therefore GFE$ (exterior angle of cyclic quadrilateral equals opposite interior angle)

$\angle GHP = \angle GPH \quad (= \angle GFE)$

(b) $GP = GH$ (equal angles are opposite equal sides in ΔGHP) 2

$FE = GH$ (opposite sides of a parallelogram are equal)

$\therefore GP = FE \quad (= GH)$

Section E (START A NEW PAGE) (15 Marks)

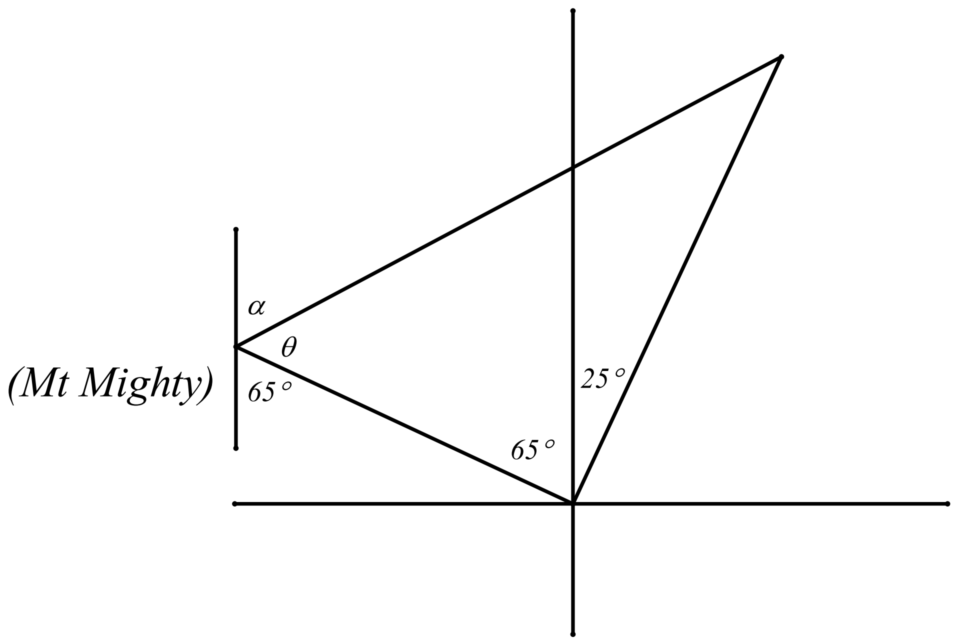
Question 1 Marks

3

$$9m^2 - 6m - 4n^2 + 4n = 9m^2 - 4n^2 - 6m + 4n$$
$$= (3m - 2n)(3m + 2n) - 2(3m - 2n)$$
$$= (3m - 2n)(3m + 2n - 2)$$

Question 2 Marks

(a) 2



(b) 2

$\angle AOB = 90°$

$AB^2 = 4.5^2 + 6.4^2$ (Pythagoras' Theorem)

$= 61.21$

$AB = 7.8\ km$

(c) 3

$\tan\theta = \dfrac{6.4}{4.5}$

$\theta = 54°53'$

$\alpha + \theta + 65° = 180°$

$\alpha = 180° - 65° - 54°53'$

$= 60°7'$

$\therefore$ bearing is $N60°E$ or $060°T$

Question 3 Marks

(a) At $A, y=0$ 1

$\therefore\ 3x-48=0$

$x=16$

A is (16,0)

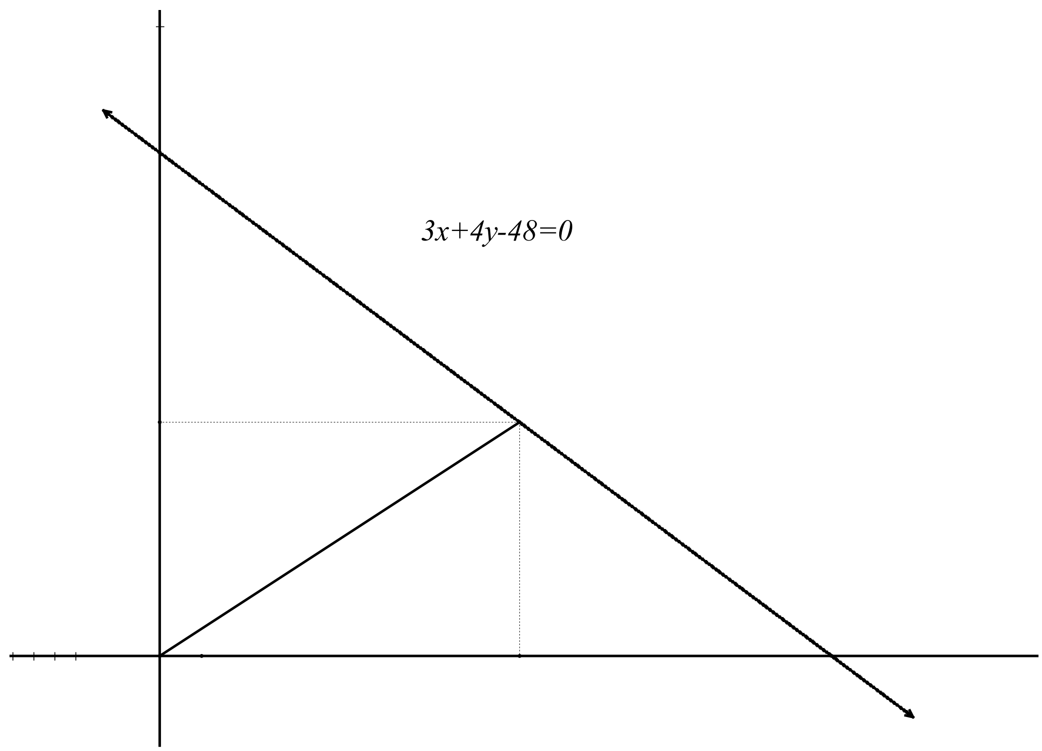
At $B, x=0$

$\therefore\ 4y-48=0$

$y=12$

B is (0,12)

(b) 4



Area of $\Delta OPA = \frac{1}{2}(16)(s) \quad u^2$

$= 8s \quad u^2$

Area of $\Delta OPB = \frac{1}{2}(12)(r) \quad u^2$

$= 6r \quad u^2$

$\frac{8s}{6r} = \frac{3}{5}$

$40s = 18r$

$s = \frac{9r}{20}$

But point $P(r,s)$ lies on the line $3x+4y-48=0$

$\therefore\ 3r+4s-48=0$

$3r+4\left(\frac{9r}{20}\right)=48$

$3r+\frac{9r}{5}=48$

$24r=240$

$r=10$

$s=\frac{9\times 10}{20}$

$s=4.5$

$\therefore P$ is $(10,4.5)$

This is the end of the examination