YEARLY EXAMINATION

YEAR 9 2009

MATHEMATICS

Time Allowed – 85 minutes plus 5 minutes Reading time

INSTRUCTIONS:

- All questions may be attempted
- Start each section on a new page
- Write your name at the top of each page
- Department of Education approved calculators and templates are permitted
- Show all necessary working
- Marks may not be awarded for untidy or carelessly arranged work
- No grid paper is to be used unless provided with the examination paper
- **Teachers: Please collect each section separately.**

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<u>Section A</u> (15 marks) Begin a Separate sheet of paper. **Marks**

1. Solve for *x*: $(2x-3)^2 = 16$ **1**

2. Factorise $8x^3 - 64$ **2**

3. Expand and simplify $4(2-3x) - 2(1-2x)$ **2**

4. Simplify $\left(\frac{64}{125}\right)^{-\frac{2}{3}}$. **2**

5. Simplify $\sqrt{8} + \sqrt{50} - \sqrt{32}$ and express the answer in the form $\sqrt{a}$. **1**

6. Express $\frac{2}{2\sqrt{3}-1}$ in its simplest form with a rational denominator. **2**

7. Fully factorise: $3a + 4b + 9a^2 - 16b^2$ **2**

8. If the axis of symmetry for $y = ax^2 + 3x + 1$ is $x = \frac{1}{2}$, find the value of a. **1**

9. Find the probability that there are at least 2 girls in a family of three children. **2**

<u>Section B</u> (15 marks) Begin a Separate sheet of paper

1. Find k if $(1\ ,-3)$ lies on the line with equation $2x - ky + 3 = 0$ **1**

2. Make M the subject of the formula in $F = \frac{MP}{M+1}$. **2**

3. Simplify $\frac{1}{x+2} - \frac{3}{x-1}$ **2**

4. *A* (-2, 3) and *B* (3, -1) are points on the Number Plane. The line *l* is perpendicular to *AB* passing through *C* (-3, 2).

   a) Calculate the distance *AB* and gradient of *AB*. Leave your answers in exact form. **2**

   b) Find the equation of line *l*, in general form. **2**

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Marks

5. Solve $3x^2 + 2x - 2 = 0$ using the quadratic formula. Leave your answers in simplified exact form. **3**

6. *O* is the centre of the circle. Find the values of *x*, *y* and *z*. Give full reasons for your answer(s). **3**

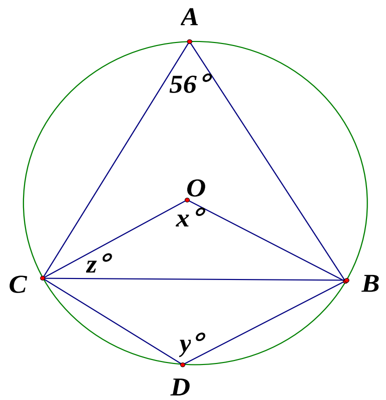
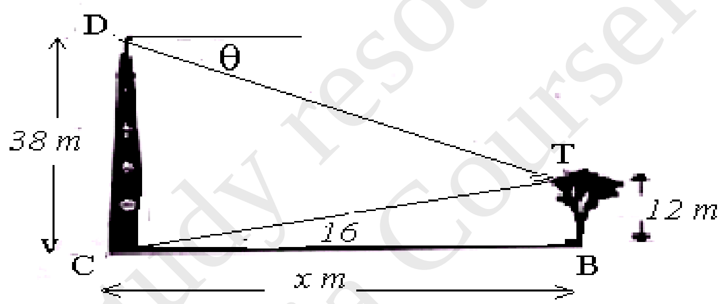


Diagram not to scale

<u>Section C</u> (15 marks) Begin a Separate sheet of paper

1. The angle of elevation to the top, *T*, of a 12 metre high tree from the base of a 38 metre high tower is 16°.



(a) Find the distance, *x* (to the nearest metre) from the base of the tree, *B*, to the base of the tower, *C*. **1**

(b) Find (to the nearest degree) the angle of depression θ^{o} to the top of the tree from the top of the tower, *D* (as indicated). **2**

2. *B* is the point (*p*,0) on the *x*-axis. A vertical line is drawn from *B* and cuts the line $3x + 2y - 18 = 0$ at the point *A*.

(a) Find the coordinates of *A* in terms of *p*. **2**

(b) The triangle bounded by *AB*, the line $3x + 2y - 18 = 0$ and the *x*-axis has an area of 12 units², find the possible coordinates of *A*. **3**

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Marks

3. A bag contains 6 red and 4 black marbles. One marble is drawn at random and not replaced. Then a second marble is drawn from the bag. Find the probability that the selection contains two marbles of different colours. **2**

4. (a) If $(x+3)$ is a factor of $P(x) = x^3 - 2x^2 + kx + 12$, find the value of k. **2**
   (b) Express $P(x)$, as a product of its factors if $(x-4)$ and $(x-1)$ are the other two factors of $P(x)$. **1**
   (c) Sketch $y = P(x)$, clearly showing all intercepts. **2**

<u>Section D</u> (15 marks) Begin a Separate sheet of paper

1. Find all real solutions to $4x^4 + 11x^2 - 3 = 0$. **3**

2. If $P(x) = 2x^3 + 11x^2 - 4$ and $A(x) = x + 1$, find $P(x) \div A(x)$ and express your answer in the form $P(x) = (x+1)(\dots\dots\dots\dots)$ + remainder. **3**

3. A builder constructs a frame (shown in the diagram) where *AC* = 6 metres and *BC* = 4 metres.
   A strut *DE* is added so that *E* is 1 metre from *C*.

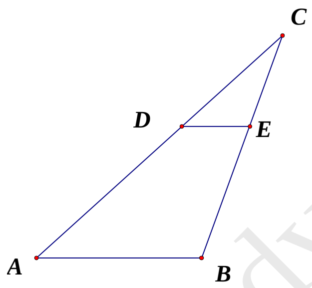

Diagram not to scale

The artist wants to know the position of *D* so that *DE* and *AB* are parallel.
   (a) If *DE* || *AB*, prove that the ΔDCE is similar to ΔACB **2**
   (b) Hence, find the position of *D* from *C*. **2**

4. A car travels for 1·1 hours at 110 km/h on a bearing of $040^o T$ from home *(H)*. Then it turns at position *X* and continues for another 1·4 hours at the same speed but on a bearing of $130^o T$ to *F*.

   (a) By drawing a diagram to represent this information, determine the distance that the car is from its starting point (to nearest kilometre). **3**
   (b) What would be its bearing from the starting point (to nearest degree)? **2**

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<u>Section E</u> (15 marks) Begin a Separate sheet of paper **Marks**

1. When $P(x) = 2x^3 + 7x^2 + ax + b$ is divided by $(x - 3)$ and $(x + 1)$, the remainders are 120 and -8 respectively. Find the values of a and b. **3**

2. A picture was cut in a circular shape and placed on top of a square frame as shown below so that the edge of the frame is just touching the circumference of the circle.

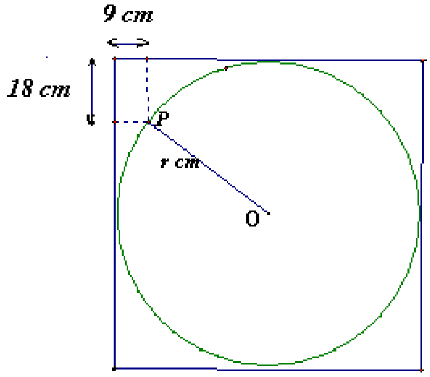

Diagram not to scale

Point P is on the edge of the circular picture piece, 18 cm from one edge of the square frame and 9 cm from the other edge of the square frame
(as shown in the diagram).

(a) Let the radius of the circular picture piece be r cm.
Show that r satisfies the equation:
$r^2 - 54r + 405 = 0$ **2**

(b) Find the radius of the circular picture piece. **2**

(c) Show that the centre of the circular picture piece from the corner of the square frame is $45\sqrt{2}$ cm. **1**

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Marks

3. An airline company Qantas recruits a maximum of 14 officers every year for its apprenticeship program. They recruit officers in areas of electrical and mechanical engineering. The company must recruit at least three in each area and the number of mechanical engineers recruited should not exceed the number of electrical engineers by more than six.

Let x represent the number of number of electrical engineers and y represent the number of mechanical engineers.

(a) Express the four constraints for this problem as linear inequalities. **2**

(b) **Detach the graph paper at the end of the exam paper to answer this question.** Complete the graph and shade the appropriate region that describes and satisfies the above conditions. **2**

If the company spends $200 on every electrical engineer recruited and $300 on every mechanical engineer recruited.

(c) What is the greatest amount the company would have to spend with these conditions and how many of each electrical and mechanical engineers would be recruited to achieve this highest spending. **3**

END OF THE PAPER

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Year 9 - Yearly Solutions - 2009

A)

1) $(2x-3)^2 = 16$

$2x - 3 = \pm 4$

$x = \frac{7}{2}$ or $x = -\frac{1}{2}$ ✓ 1

2) $8x^3 - 64 = 8(x-2)(x^2+2x+4)$ ✓ ✓ 2

3) $4(2-3x) - 2(1-2x)$

$= 8 - 12x - 2 + 4x$ ✓

$= 6 - 8x$ ✓ 2

4) $\left(\frac{64}{125}\right)^{-2/3} = \left(\frac{125}{64}\right)^{2/3} = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$ or $1\frac{9}{16}$ ✓ ✓ 2

5) $\sqrt{8} + \sqrt{50} - \sqrt{32} = 2\sqrt{2} + 5\sqrt{2} - 4\sqrt{2}$

$= 3\sqrt{2} = \sqrt{18}$ ✓ 1

6) $\frac{2}{2\sqrt{3}-1} \times \frac{2\sqrt{3}+1}{2\sqrt{3}+1} = \frac{4\sqrt{3}+2}{11}$ ✓ ✓ 2

7) $3a+4b+9a^2-16b^2$

$3a+4b + (3a+4b)(3a-4b)$

$(3a+4b)(1+3a-4b)$ ✓ ✓ 2

8) Axis of symmetry $= \frac{-b}{2a}$

$\frac{-3}{2a} = \frac{1}{2}$

$-6 = 2a$

$a = -3$ ✓ 1

9) $(GGB)+(GBG)+(BGG)+(GGG)$

✓ $\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$

$\frac{4}{8}$

$= \frac{1}{2}$ ✓ 2

B)

1) $2x - ky + 3 = 0$

$2(1) - k(-3) + 3 = 0$

$2 + 3k + 3 = 0$

$3k = -5$

$k = -5/3$ ✓ 1

2) $F = \frac{MP}{M+1}$

$F(M+1) = MP$

$FM + F = MP$

$F = MP - FM$

$F = M(P-F)$

$\therefore M = \frac{F}{P-F}$ ✓ ✓ 2

3) $\frac{1}{x+2} - \frac{3}{x-1}$

$= \frac{x-1-3(x+2)}{(x+2)(x-1)}$

$= \frac{x-1-3x-6}{(x+2)(x-1)}$ ✓

$= \frac{-2x-7}{(x+2)(x-1)}$ ✓ 2

4) a) $A = (-2,3)$

$B = (3,-1)$

$C = (-3,2)$

Distance (AB) $= \sqrt{(-1-3)^2 + (3--2)^2}$

$= \sqrt{(-4)^2 + 5^2}$

$= \sqrt{41}$

$\therefore$ distance AB is $\sqrt{41}$ units ✓ 1

Gradient $= \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-1-3}{3--2} = \frac{-4}{5}$ ✓ 1

b) Gradient of perpendicular line is $5/4$

$\therefore$ eqn is $y - y_1 = m(x-x_1)$

$= y - 2 = 5/4(x+3)$ ✓

continued

$= y - 2 = \frac{5x}{4} + \frac{15}{4}$

$4y - 8 = 5x + 15$

$\therefore$ eq^n is $\underline{4y - 5x - 23 = 0}$ ✓

or $\underline{5x - 4y + 23 = 0}$ 2

5) $a = 3$, $b = 2$ and $c = -2$.

Quadratic formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ✓

$= \frac{-2 \pm \sqrt{4 + 24}}{6}$

$= \frac{-2 \pm \sqrt{28}}{6}$ ✓

$= \frac{-2 \pm 2\sqrt{7}}{6}$

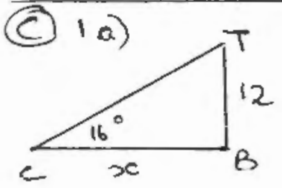
$= \frac{-1 \pm \sqrt{7}}{3}$ ✓ 3

6) $\underline{x = 112^\circ}$ (angle at centre of circle is twice the angle at circumference). ✓

$\underline{y = 124^\circ}$ (Opposite angles in cyclic quadrilateral are supplementary). ✓

$\underline{z = 34^\circ}$ (Base angles in an isosceles triangle) are equal ✓ 3

C 1a)



$\tan 16 = \frac{12}{x}$

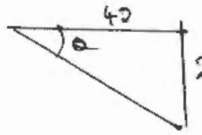
$x = \frac{12}{\tan 16}$

$= 41.8489733$

$= \underline{42}$ (nearest m) ✓ 1

Distance is 42 m

b)

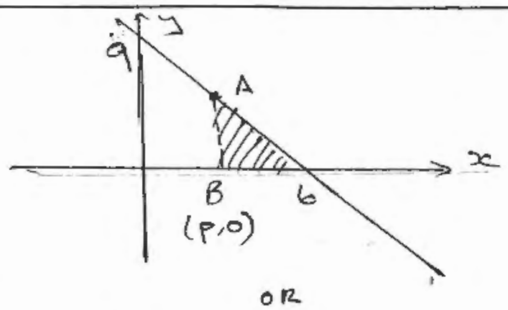


$\tan \theta = \frac{26}{42}$ ✓

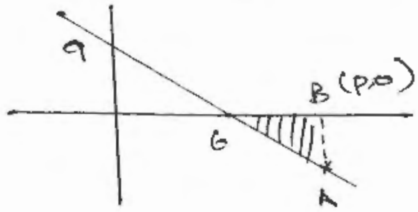
$\angle\theta = 31.75948008$

$= \underline{32^\circ}$ (nearest degree) ✓ 2

2)



OR



a) Coordinates of A (x, y)

$x = p$ ✓

$y =$

$3p + 2y - 18 = 0$

$2y = 18 - 3p$

$y = 9 - \frac{3}{2}p$ or $\frac{18 - 3p}{2}$ ✓

$\therefore$ Coordinates of A is $\left(p, \frac{18-3p}{2}\right)$ 2

b) Area $= \frac{1}{2}bh$

$12 = \frac{1}{2}(6 - p)\left(\frac{18 - 3p}{2}\right)$

$48 = (6 - p)(18 - 3p)$

$48 = 108 - 18p - 18p + 3p^2$ ✓

$0 = 3p^2 - 36p + 60$

$0 = 3(p^2 - 12p + 20)$

$0 = 3(p - 10)(p - 2)$

$\therefore p - 10 = 0$ or $p - 2 = 0$

$\therefore p = 10$ $\quad p = 2$ ✓

$\therefore$ Coordinates of A could be

when $p = 2$ $\quad x = 2$

$y = \frac{18 - 3(2)}{2} = 6$

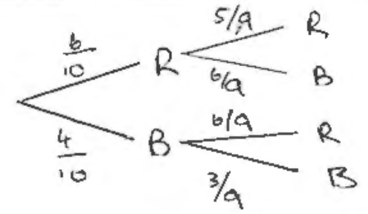
$\therefore$ one point is $\underline{(2, 6)}$ ✓

when $p = 10$ $\quad x = 10$

$y = \frac{18 - 3(10)}{2} = \frac{-12}{2} = -6$

$\therefore$ other possibility is $\underline{(10, -6)}$ 3

3



$p(RB) + p(BR) = \left(\frac{6}{10} \times \frac{4}{9}\right) + \left(\frac{4}{10} \times \frac{6}{9}\right)$ ✓

$= \frac{24 \times 2}{90}$

$= \frac{48}{90}$ or $\frac{8}{15}$ ✓ 2

4.
a) $P(-3) = -3^3 - 2(-3^2) + (-3k) + 12 = 0$ ✓

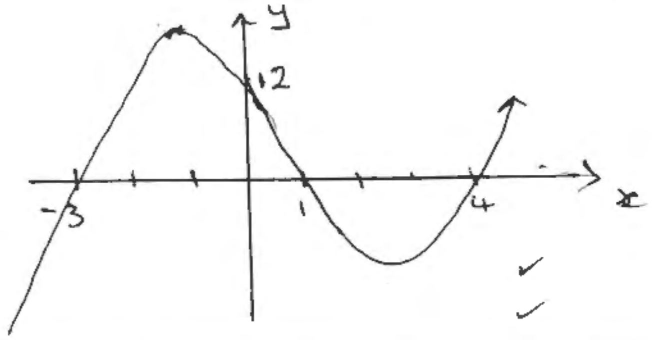
$= -27 - 18 - 3k + 12 = 0$

$= -33 - 3k = 0$

$K = -11$ ✓ 2

b) $P(x) = (x+3)(x-4)(x-1)$ ✓ 1

c)



✓
✓ 2

D 1) $4x^4 + 11x^2 - 3 = 0$

let $m = x^2$

$4m^2 + 11m - 3 = 0$ ✓

$4m^2 + 12m - m - 3 = 0$

$4m(m+3) - 1(m+3) = 0$

$(4m-1)(m+3) = 0$

$m = \frac{1}{4}$ or $m = -3$ ✓

$\therefore x^2 = \frac{1}{4}$ or $x^2 = -3$

No real solutions possible.

$x = \pm\sqrt{\frac{1}{4}}$

$x = \pm\frac{1}{2}$ ✓ 3

2) $x+1 \,\overline{)\, 2x^3 + 11x^2 - 4}$ quotient $2x^2 + 9x - 9$ ✓

$2x^3 + 2x^2$

$9x^2 + 0x$ ✓ process

$9x^2 + 9x$

$-9x - 4$

$-9x - 9$

5 ✓

$\therefore P(x) = (x+1)(2x^2 + 9x - 9) + 5$ 3

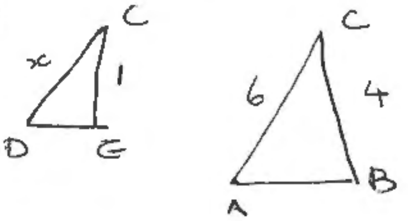
3a) $\angle DAB = \angle CDE$ (corresponding angles are equal as $AB \parallel DE$)

$\angle ABE = \angle DEC$ (As above) ✓

$\angle DCE = \angle ACB$ ($\angle C$ is common) ✓

$\therefore \triangle ACB \parallel\!| \triangle DCE$ (equiangular. 2

b)



$\frac{DC}{AC} = \frac{CE}{BC}$ (corresponding sides of similar triangles in same ratio) ✓

$\frac{x}{6} = \frac{1}{4}$

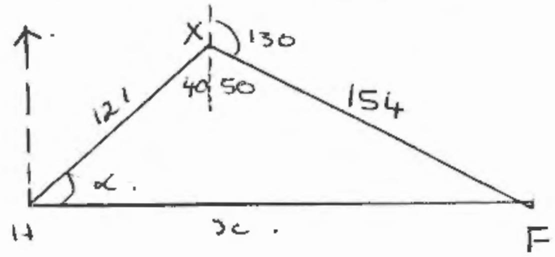
$x = \frac{6}{4}$

$\therefore$ D is 1.5 m from C ✓ 2

4. Distance = speed × time

$1.1 \times 110 = 121$ km

$1.4 \times 110 = 154$ km ✓



a) $\angle HXF = 90$ ✓

By pythagoras

$x^2 = 121^2 + 154^2$

$= 38357$

$x = \sqrt{38357}$

$= 195.8494 32$ ✓

$= 196$ (nearest km/h)

∴ distance is 196 km

3

b) $\tan \alpha = \frac{154}{121}$ ✓

$\angle \alpha = 51.8427734$

$= 52°$ (nearest degree)

∴ Bearing is $(40+52)$ 092°T or S 88° E ✓

2

Ⓔ 1. $P(x) = 2x^3 + 7x^2 + ax + b$

✓ $P(3) = 54 + 63 + 3a + b = 120$

$3a + b = 3$ ---- ①

$P(-1) = -2 + 7 - a + b = -8$

$-a + b = -13$ ---- ②

② − ① $3a + b = 3$ ✓

$-a + b = -13$

$4a = 16$

$a = 4$

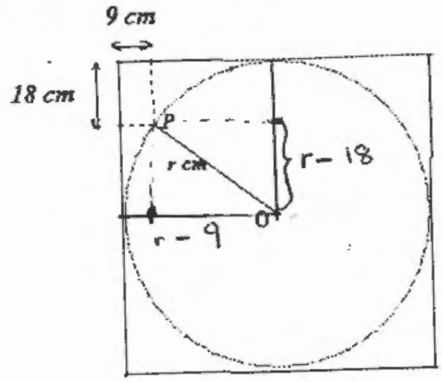
Sub in ① $3(4) + b = 3$

$b = -9$

∴ $a = 4$ and $b = -9$ ✓

3

2a)



$r^2 = (r-18)^2 + (r-9)^2$ ✓

$r^2 = r^2 - 36r + 324 + r^2 - 18r + 81$

$r^2 = 2r^2 - 54r + 405$

$0 = r^2 - 54r + 405$ ✓

2

b) $(r - 45)(r - 9) = 0$ ✓

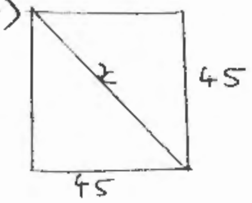
$r = 45$ or $r = 9$

but $r > 9$

∴ $r = 45$ cm ✓

2

c)



By Pythagoras

$x^2 = 45^2 + 45^2$

$= 2025 + 2025$ ✓

$= 2025 \times 2$

$x = \sqrt{2025 \times 2}$

$= 45\sqrt{2}$

∴ Distance is $45\sqrt{2}$ cm

1

3a $x + y \le 14$

$x \ge 3$ ½ mark each

$y \ge 3$

$y \le x + 6$

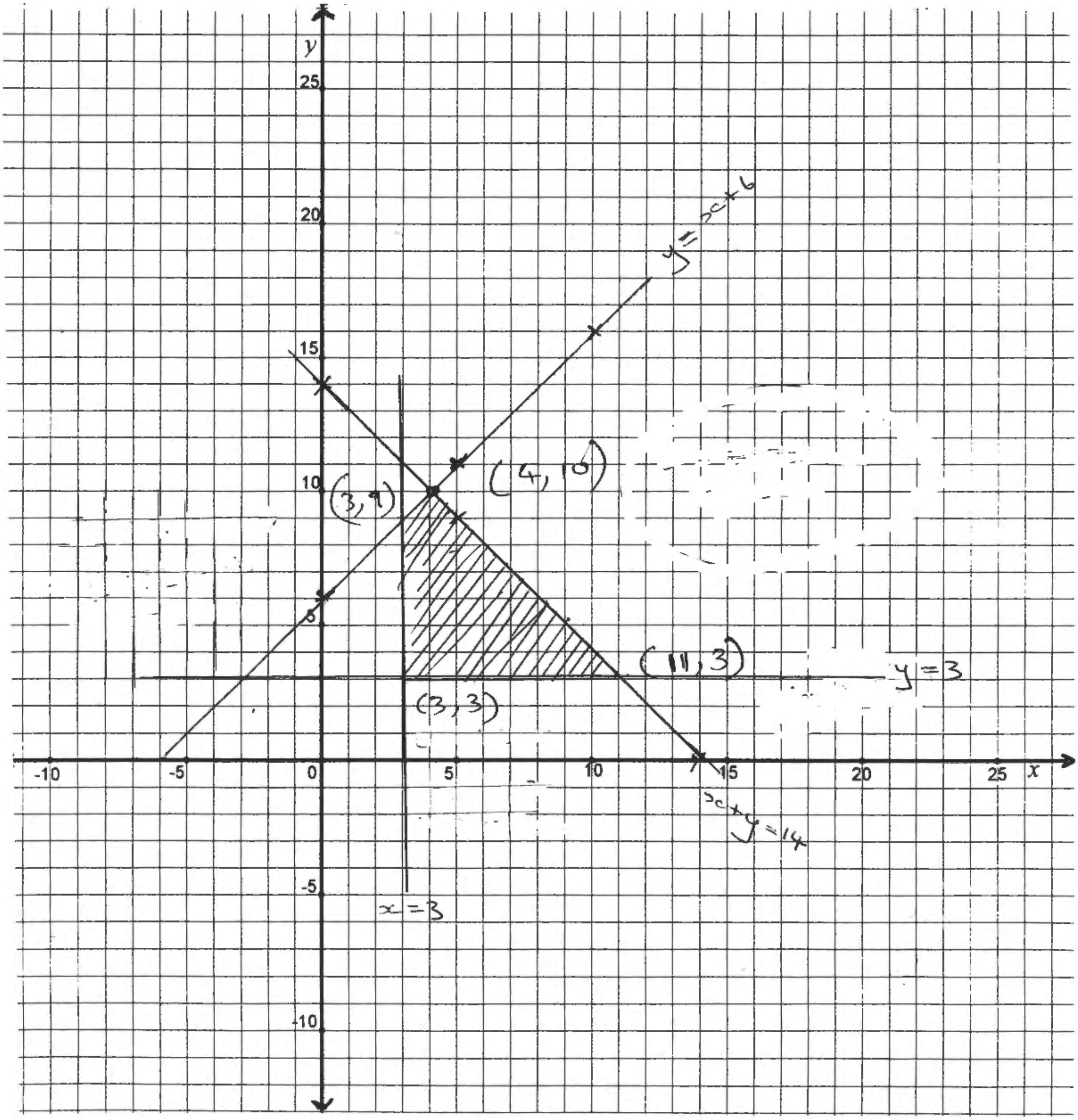
2

Section E Question 3 (b)

Name..

Class ....

Remove this graph and attach to your solutions for section E. Q3 (b)



✓✓ 2 marks

c) spending $= 200x + 300y$

Point (3,3) $= 200 \times 3 + 300 \times 3 = \$1500$

Point (11,3) $= 200 \times 11 + 300 \times 3 = \$3100$

Point (4,10) $= 200 \times 4 + 300 \times 10 = \$3800$ ✓

Point (3,9) $= 200 \times 3 + 300 \times 9 = \$3300$

$\therefore$ highest spending is \$3800 ✓
recruit 4 electrical and
10 mechanical engineers ✓ 3

The end.