YEARLY EXAMINATION

YEAR 9 2011

MATHEMATICS

Time Allowed: 85 minutes plus 5 minutes reading time.

Instructions:

- All questions may be attempted
- Start each section on a new page
- Write your name at the top of each page
- **Write** in **Pen** and draw diagrams in **Pencil**
- Board-approved calculators and templates are permitted
- Show all necessary working
- Marks may not be awarded for untidy or carelessly arranged work
- No grid paper is to be used unless provided with the examination paper
- **Teachers: Please collect each section separately**

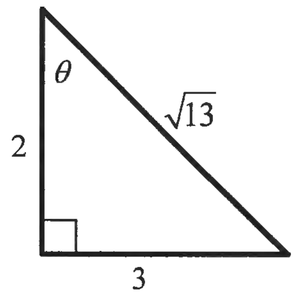
Section A (20 Marks) Start a new page

(1) Solve for y: $17+6y=35$. **1**

(2) Evaluate $\sin 54°$, leaving your answer correct to 2 decimal places. **1**

(3) Expand $\left(\frac{3}{2}m-n\right)\left(\frac{3}{2}m+n\right)$. **1**

(4) Using the diagram,write down the value of $\tan\theta$. **1**



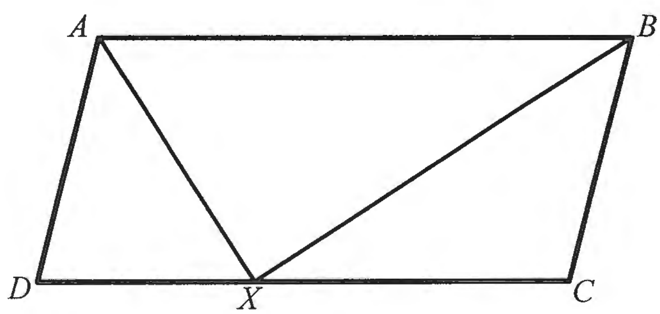
(5) Write down the gradient of the line $2x+y-7=0$. **1**

(6) Express 4^{-3} as a simple fraction without indices. **1**

(7) Find x given that $\sqrt{18}+\sqrt{8}=\sqrt{x}$. **1**

(8) In the diagram below, $ABCD$ is a parallelogram. **3**
BX bisects $\angle ABC$ and AX bisects $\angle BAD$.
Prove $\angle BXA$ is a right angle.

Diagram not drawn to scale



(9) Write down the equation of the vertical line through the point $(-2, 3)$. **1**

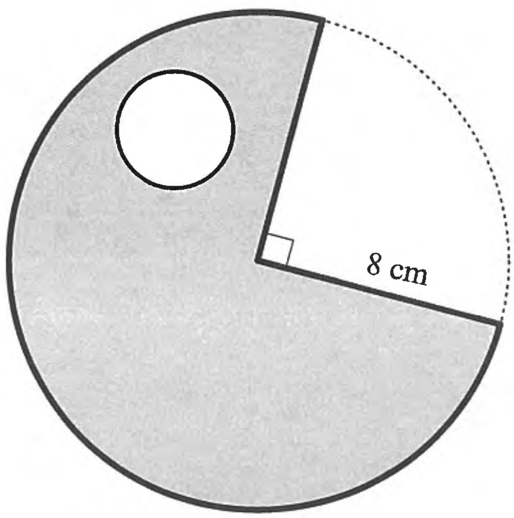
(10) Factorise (i) $x^2 - 64y^2$ **1**

(ii) $2x^2 + 7x - 15$ **1**

(11) Solve the following equations simultaneously. **2**

$$5x + 2y = 3$$
$$3x - 4y = -19$$

(12) The image below shows a computer game character called Pacman. **2**
The image is created by removing a circle of radius 1 cm and a quarter-circle from a large circle of radius 8 cm as shown below.
Find the exact area of the shaded area of Pacman, leaving your answer in terms of π.



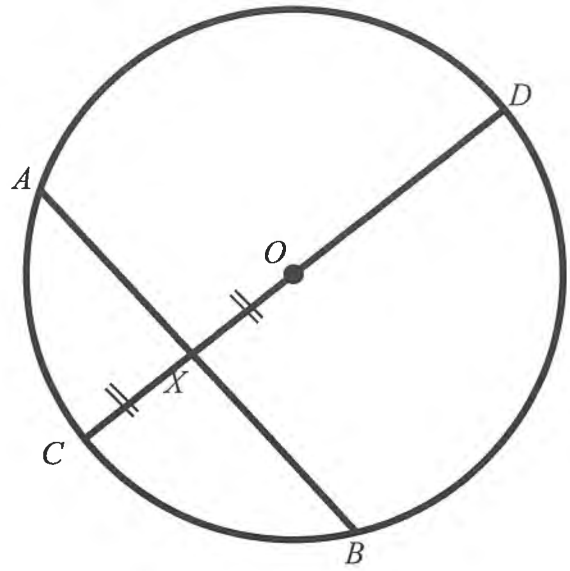
(13) Phillip is 24 years older than his son George. **3**
Three years from now, Phillip will be $2\frac{1}{2}$ times as old as George.
How old are both right now?

End of Section A

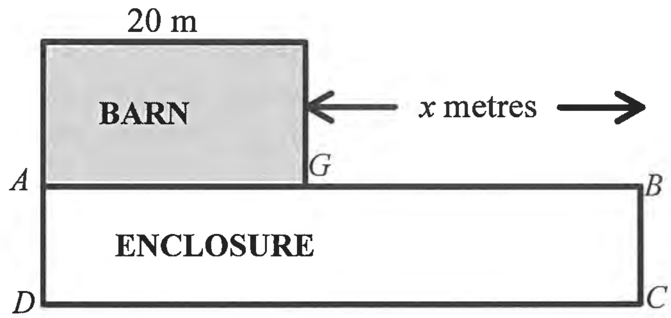
Section B (20 Marks) **Start a new page**

(1) The line l has equation $3x-4y+11=0$. **2**
Find the equation of the line through $(6,-1)$ which is perpendicular to l.

(2) In the diagram below, the chord AB and diameter CD intersect at X. **3**
$CX = XO$, $CD = 8$ cm and $AX = 3$ cm.
Find, stating all reasons, the length of XB.



(3) A farmer wishes to enclose the rectangular area $ABCD$, as shown in the diagram below.



He can use the side of the barn as part of the enclosure, but must fence the other sections. The barn is 20 m long. The farmer has 100 m of fencing available altogether. Let the distance BG be x metres.

(i) Copy the diagram onto your answer sheet and find expressions for the lengths of CD and BC. **2**

(ii) Hence show that the area of the enclosure is given by $A = 800 + 20x - x^2$. **3**

(iii) Find the value of x that will give the maximum area of the enclosure. **2**

(4) Find p if $x-2$ is a factor of x^3+px^2+x+6 **2**

(5) By completing the square, solve $x^2+6x-1=0$.
Leave your answers correct to three decimal places. **2**

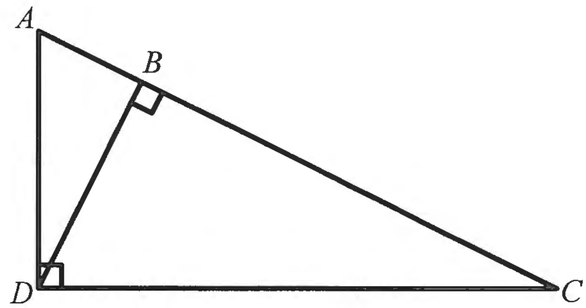
(6) Sketch the graph of the polynomial $y=(x-2)(x+1)^2$.
Clearly indicate the intercepts with the coordinate axes. **2**

(7) What are the coordinates of the vertex of the parabola $y=(x+1)(x-3)$? **2**

End of Section B

Section C (20 Marks) **Start a new page**

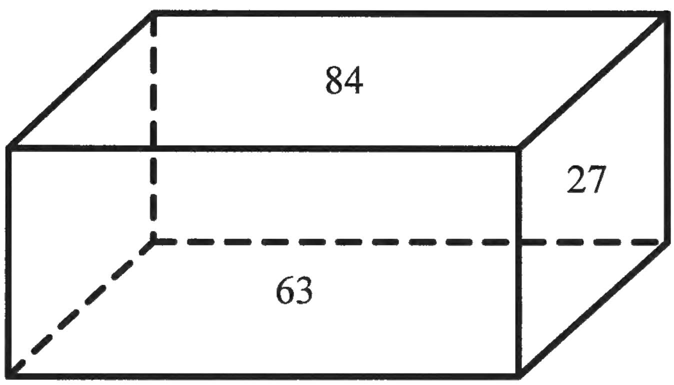
(1) In the diagram below, $AD = 4$cm and $AC = 7$ cm. $AD \perp DC$ and $DB \perp AC$.



(i) Prove that $\Delta ABD \parallel\!| \Delta ADC$. **3**

(ii) Find the length of AB, with reasons. **1**

(2) The diagram below shows a rectangular prism. **2**
The areas of the top, front and side faces are 84 cm^2, 63 cm^2 and 27 cm^2 respectively.



Find the volume of the prism.

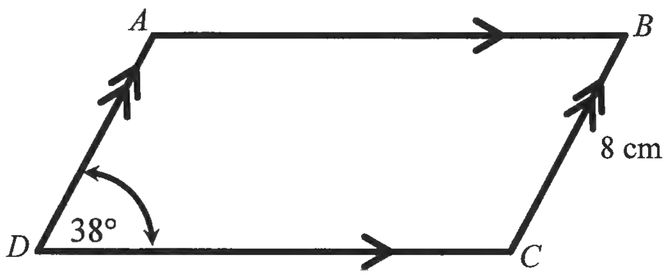
(3) Fully factorise

(i) $a^2b - a^2c + ac^2 - ab^2 + b^2c - bc^2$. **2**

(ii) Factorise $3^{n+1} + 3^n$. **2**

Hence, write $\dfrac{3^{501} + 3^{500}}{4}$ as a power of 3.

(4) The diagram below shows parallelogram $ABCD$ with $\angle ADC = 30°$ and $DC = 8$cm.



(i) Find, with reasons, the size of $\angle ABC$. **1**

(ii) If the diagonals AC and BD are perpendicular, find the perimeter of $ABCD$. **2**
Give reasons for your answer.

(5) The stem-and-leaf plot below shows the mass in kilograms of the students in a class. **1**
Determine the median mass of the students.

4	6	7	9					
5	0	2	3	5	5	6	8	9
6	1	2	4	4	5	7	8	
7	0	3						

(6) Basket ball player Ray Allen scored the following points in 13 games.
The data is displayed in ascending order

12, 15, 16, 16, 16, 19, 21, 24, 26, 27, 29, 29, 36

(i) Find the mode. **1**

(ii) Find the median. **1**

(iii) Find the upper and lower quartiles. **1**

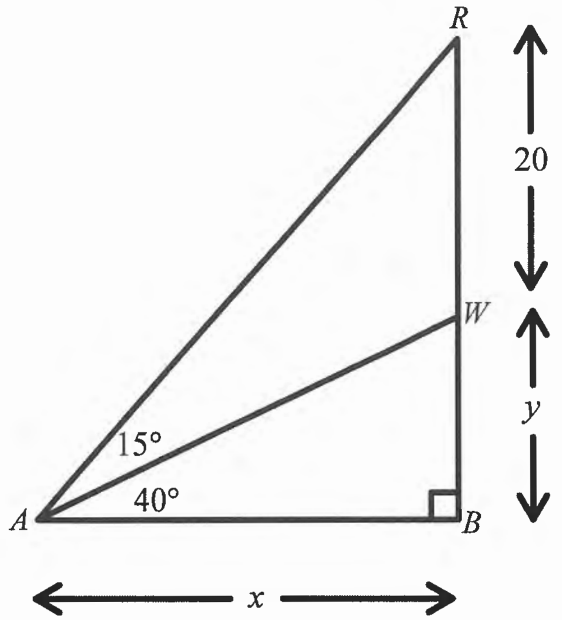
(iv) Draw a box plot to display this information. **2**

(v) Calculate the mean. **1**

End of Section C

Section D (20 Marks) Start a new page

(1)



From a point A, Prince Harry can see Rapunzel standing at the window, W, of her castle at an angle of elevation of $40°$.
Rapunzel then ascends 20 m to the roof of the castle, R, and her angle of elevation from Prince Harry is now $55°$.
Let AB be x metres and BW be y metres.

(i) Explain why $x = y\tan 50°$. **2**

(ii) By considering ΔABR, find another expression for x. **2**

(iii) Calculate the height of the castle correct to 1 decimal place. **6**

(2) The numbers of units of chemicals A, B and C in a bag of Fertilizer X and Fertilizer Y are given in the table below.

Chemicals	A	B	C
Fertilizer X	1	1	3
Fertilizer Y	2	1	1

A farmer wants to obtain at least 20 units of A, 16 units of B and 24 units of C to fertilize a piece of land.

If the cost per bag of X and Y are \$24 and \$12 respectively, find the minimum costs of fertilization. **10**

End of Section D

Section E (20 Marks) Start a new page

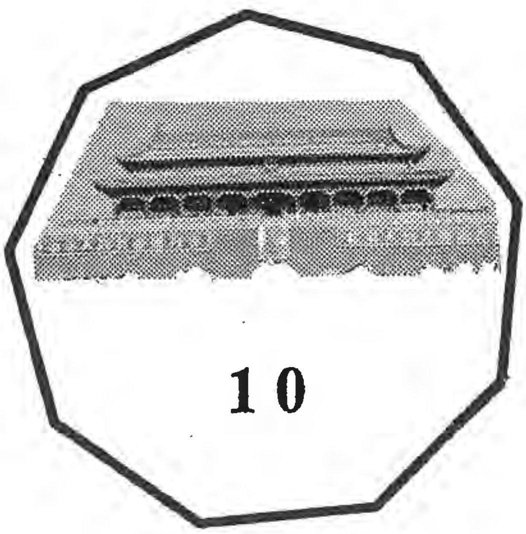
(1) (i) By expanding, show that: $\left(x-\frac{1}{x}\right)^3 = x^3 - 3x + \frac{3}{x} - \frac{1}{x^3}$. **2**

(ii) If $x - \frac{1}{x} = 5$ and without calculating the value of x,

(a) Find the value of $x^3 - \frac{1}{x^3}$. **2**

(b) Find the value of $x^4 + \frac{1}{x^4}$. **2**

(2) The new Peoples' Republic of China Ten Dollars coin is to be in the shape of a regular nonagon,with each side 8 mm in length.



(i) Find the size of each interior angle. **2**

(ii) The area, A_n, of a regular n-sided polygon with side length x units is given by the formula **2**

$$A_n = \frac{nx^2}{4\tan\left(\frac{180^\circ}{n}\right)}$$

Find the area of the coin, leaving your answer to the nearest mm^2.

(3) The equation $x^2 = (y+1)^2 - y^2$, where x and y are both positive integers has an infinite number of solutions.

(i) One solution is $x = 3$, $y = 4$. Find six other solutions. **3**

(ii) Write down six solutions to the equation $x^2 + y^2 = z^2$, where x, y and z are all positive integers with no factors in common and where $x < y < z$. **4**

(iii) Show that there are no solutions to the equation in (ii) under the conditions specified there and with y and z differing by exactly 3. **3**

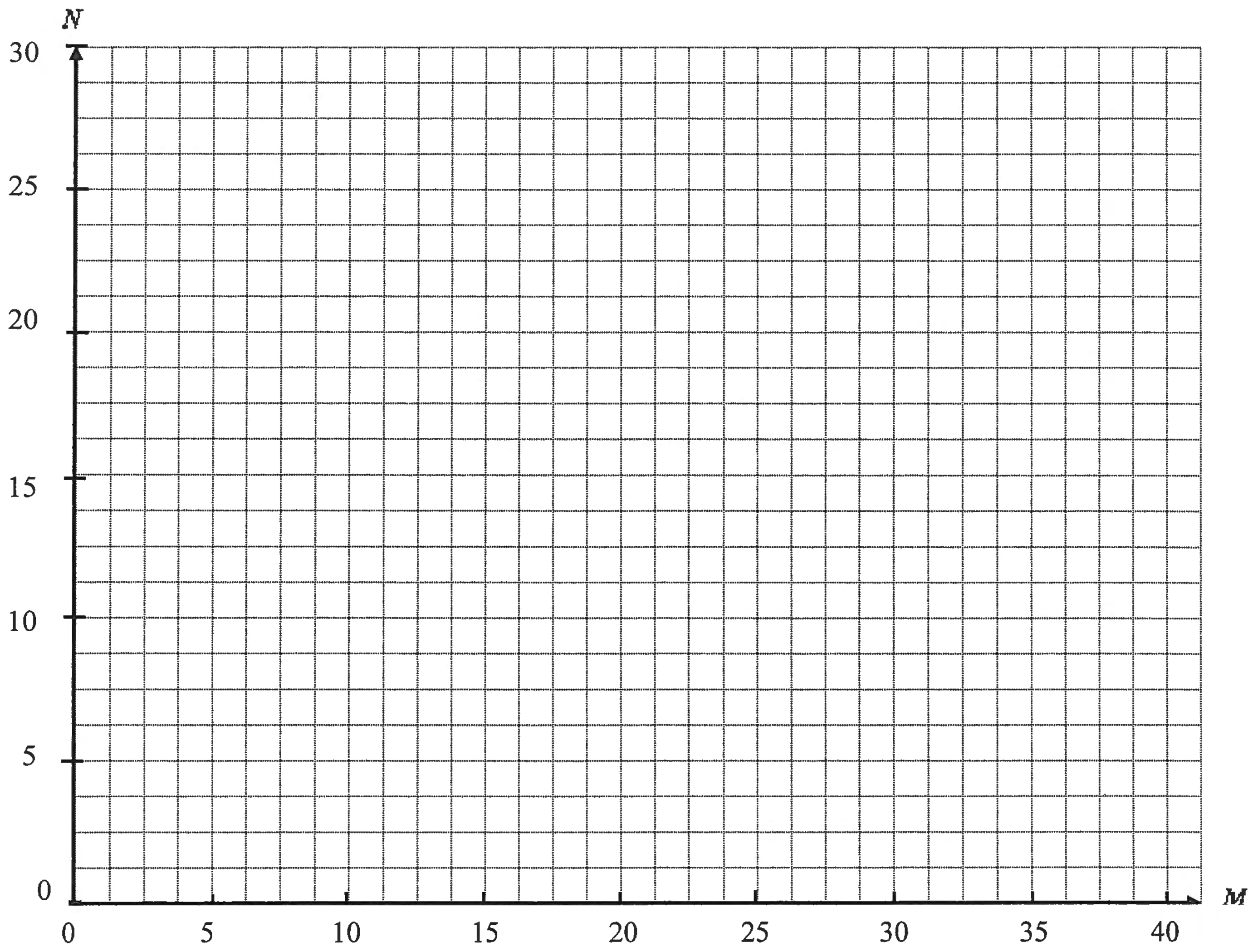
End of paper

Section D **Question 2**

REMOVE THIS PAGE AND STAPLE TO **SECTION D** OF YOUR ANSWER BOOKLET

Name: ______________________________ **Year 9** _____

Q2. To be done on the grid below.



1/ $17+6y=35$
$6y=18$
$y=3$

2/ $\sin 54^\circ = 0.81$ (2DP)

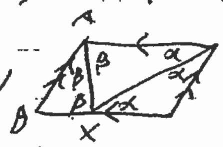
3/ $\left(\frac{3m}{2}\right)^2 - n^2 = \frac{9m^2}{4} - n^2$

4/ $B^2 = 13-4$ $\therefore B=3$
$\therefore \tan\theta = 2/3$

5/ $y=-2x+5$
$\therefore$ Gradient is -2

6/ $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

7/ $3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$

8/ 

$2\alpha + 2\beta = 180$
(co-int. ∠'s)
$\therefore \alpha+\beta = 90$, $\angle AXB = 180-(\alpha+\beta) = 90$

9/ Line is $x=-2$

10/ (i) $x^2 - (8y)^2 = (x+8y)(x-8y)$
(ii) $2x^2+10x-3x-15$
$2x(x+5)-3(x+5)$
$=(2x-3)(x+5)$

11/ $10x+4y=6$ — ①
$3x-4y=-19$ — ②

① + ② $\Rightarrow 13x=-13$
$x=-1$

Sub into ②
$4y=-3+19$
$y=4$
$\therefore (x,y)=(-1,4)$

12/ $\pi(8)^2 \times \frac{3}{4} - \pi(1)^2$
$= 3\times 16\pi - \pi = 47\pi\ m^2$

13/ Let George be x now and Philip y.
Now $y=x+24$ ①
In 3 years $y+3 = \frac{5}{2}(x+3)$
$\therefore 2y+6 = 5x+15$
$2y=5x+9$
Sub ① $2x+48=5x+9$
$39=3x$
$x=13$
Sub ① $\therefore y=37$
$\therefore$ George is 13 and Philip 37 now.

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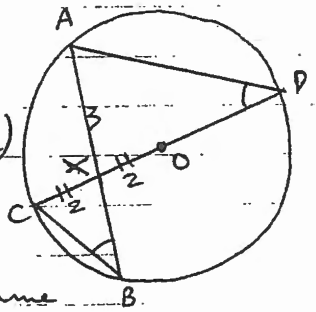
1/ (i) $m = 3/4$ (all parallel lines have same gradient)

ii) Line of form $4x+3y=k$
goes thro' $(6,-1)$
$\therefore 24-3 = k$
$k = 21$
$\therefore$ Line is $4x+3y-21=0$

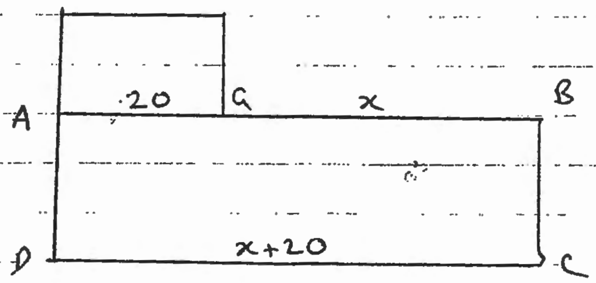
2/



a) $CD = 8$ (given)
$\therefore CO = 4$ (Radius $= d/2$)
In $\triangle$s AXD, CXB
$\angle XBC = \angle XDA$
(Angles standing on same arc are equal)
$\angle XCB = \angle XAD$ (Similarly)
$\therefore \triangle AXD \,|||\, \triangle CXB$ (Equiangular)
$\therefore \frac{XD}{XB} = \frac{AX}{CX}$ (Corresponding sides of similar triangles are in fixed ratio.)

$\therefore \frac{6}{BX} = \frac{3}{2}$
$\therefore BX = 4$ (cm)

3/



ii) $CD = x+20$

$2BC + 2x + 20 = 100$
$\therefore BC = 40 - x$

iii) $A = (x+20)(40-x)$
$= 40x + 800 - x^2 - 20x$
$= 800 + 20x - x^2$

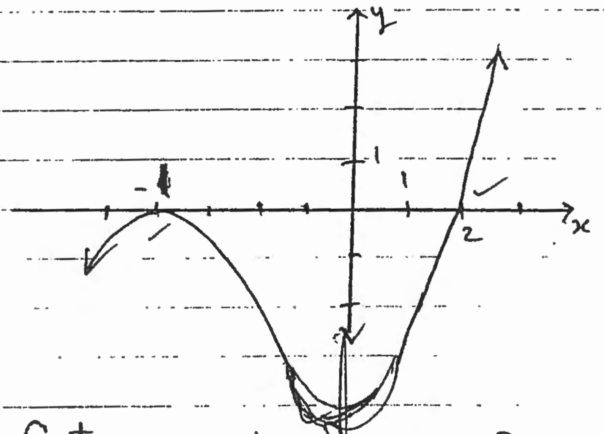
iv) $A = -(x^2 - 20x - 800)$
$= -((x-10)^2 - 100 - 800)$
$= -(x-10)^2 + 900$

$\therefore$ Max area when $x = 10$.

4/ By factor theorem
$2^3 + p2^2 + 2 + 6 = 0$
$\therefore 4p = -8-8$
$p = -4$

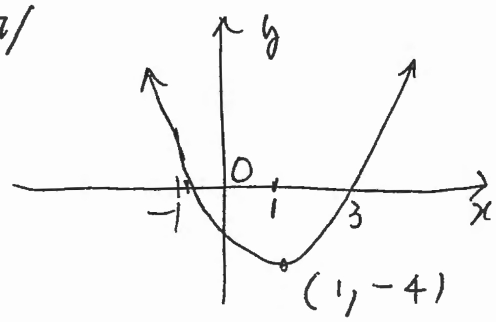
5/ $x^2 + 6x - 1 = 0$
$(x+3)^2 - 9 - 1 = 0$
$(x+3)^2 = 10$
$x+3 = \pm\sqrt{10}$
$x = -3 \pm \sqrt{10}$
$\therefore x = -6.162$ or 0.162

6/



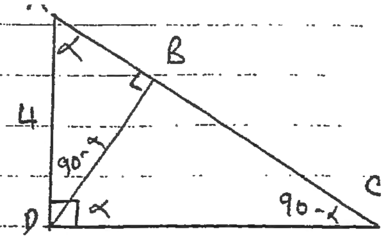
Cuts y axis when $x = 0$
i.e. $y = -2$
$\therefore$ Cuts at $(0, -2)$

7/



(C)

1/



In Δs ABD, ACD

Let $\angle BAD = \alpha$

$\angle BCD = 90° - \alpha$ (Angles in ΔACD add to 180°)

$\angle ACD = 90° - \alpha$ (Same angle)

$\angle ADB = 90° - \alpha$ (Angles in ΔABD add to 180°)

$\angle ADC = 90°$ (given)

$\angle ABD = 90°$ (given)

$\therefore \Delta ABD \,|||\, \Delta ADC$ (Equiangular)

$\therefore \frac{AB}{AD} = \frac{AD}{AC}$ (Corresponding sides in similar triangles are in constant ratio)

$\therefore \frac{AB}{4} = \frac{4}{7}$

$AB = 16/7$ cm.

2/ Let dimensions be a, b, c so that

$ab = 84$

$bc = 63$

$ca = 27$

Divide last two

$b/a = 63/27 = 7/3$

$\therefore b = \frac{7a}{3}$

Sub into first

$\frac{7a^2}{3} = 84$

$a^2 = 36$

$a = 6$ ($a > 0$, length)

$\therefore b = 14$

Also $c = 4\tfrac{1}{2}$ (from 3rd eqn).

$\therefore$ Volume $= 6 \times 14 \times \frac{9}{2}$

$= 378$ cm³

3/ i) $a^2(b-c) + a(c^2 - b^2) + bc(b-c)$

$= (b-c)(a^2 - a(b+c) + bc)$

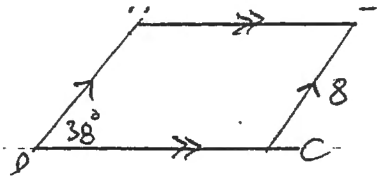
$= (b-c)(a^2 - ab - ac + bc)$

ii) $3^{n+1} + 3^n = 3^n(3+1)$

$= 4 \times 3^n$

$\therefore \frac{3^{501} + 3^{500}}{4} = \frac{4 \times 3^{500}}{4} = 3^{500}$

4/



i) $\angle ABC = 38°$ (Opposite angles in parallelogram are equ

ii) Diagonals perpendicular means ABCD is a rhombus. A rhombus has all sides equ

$\therefore$ Perimeter = 32 cm

5/ Median 57 kg.

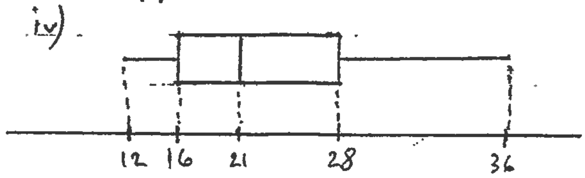
6/ i) Mode = 16

ii) Median = 21

iii) Lower Quartile = 16

Upper Quartile = 28

iv)



v) Mean is 22

(2)

1/ i) $\frac{y}{x} = \tan 40^\circ = \cot 50^\circ$

$\therefore \frac{x}{y} = \tan 50^\circ \qquad x = y\tan 50^\circ$

ii) $\tan 55^\circ = \frac{y+20}{x}$

$x = \frac{(y+20)}{\tan 55^\circ} = (y+20)\cot 35^\circ$

iii) $\therefore y\tan 50 = (y+20)\cot 35^\circ$

$y = \frac{20\cot 35^\circ}{\tan 50^\circ + \cot 35^\circ}$

$= 10.902...$

$= 30.9\text{m}$ (nearest 1

(D)

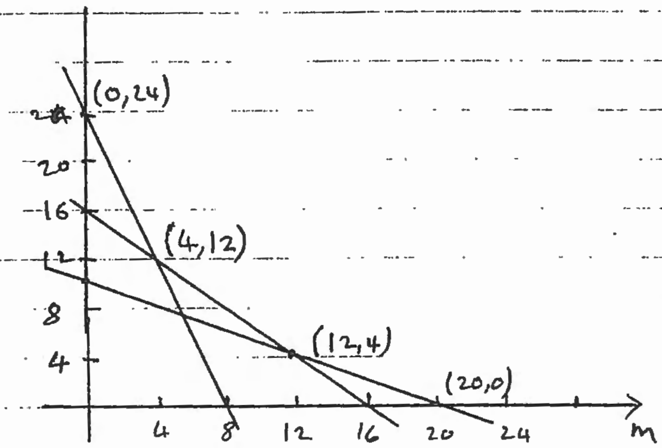
2/ Let him buy m bags of X and n bags of Y.

$\therefore \quad m + 2n \geq 20 \quad (A)$

$m + n \geq 16 \quad (B)$

$3m + n \geq 24 \quad (C)$

Cost is $24m + 12n$ ($).



Evaluate cost at each corner.

m, n	24m + 12n
(0, 24)	$288
(4, 12)	$240
(12, 4)	$336
(20, 0)	$480

Minimum cost is $240

(with 4 bags of X and 12 bags of Y).

$\left(x-\frac{1}{x}\right)^3 = x^3 - 3x + \frac{3}{x} - \frac{1}{x^3}$

(1)

ii) $x - \frac{1}{x} = \sqrt{3} \quad (>0)$

$\therefore$ a) $x^3 - \frac{1}{x^3} = \left(x-\frac{1}{x}\right)^3 + 3\left(x-\frac{1}{x}\right)$ from (i)

$= 3\sqrt{3} + 3\sqrt{3}$

$= 6\sqrt{3}$

b) $\left(x-\frac{1}{x}\right)^4 = x^4 - 4x^2 + 6 - \frac{4}{x^2} + \frac{1}{x^4}$

$\therefore x^4 + \frac{1}{x^4} = 3^2 + 4\left(x^2 + \frac{1}{x^2}\right) - 6$

$x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$

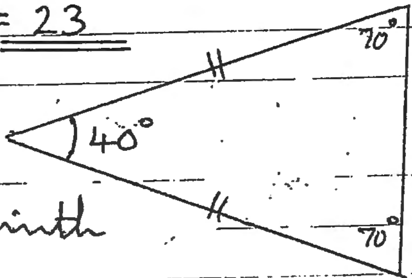
$= 5$

$\therefore x^4 + \frac{1}{x^4} = 9 + 4 \times 5 - 6$

$= 23$

(2)

(3) i)



One ninth

Nine such triangles make up the nonagon.

$\therefore$ Sum $= 9 \times 140° = 1260°$

ii) Each interior angle is $1260/9$
$= 140°$

iii) $A_9 = \frac{9 \times 8^2}{4 \tan 20°} = 395.6367...$

Area is $396\,mm^2$ to nearest mm^2

(3)

(i) The equation simplifies to $x^2 = 2y + 1$, thus x must be odd.

x	5	7	9	11	13	15
x^2	25	49	81	121	169	225
y	12	24	40	60	84	112

(ii) Six solutions follow immediately from part (i) with $z = y + 1$. Thus;

x	5	7	9	11	13	15
y	12	24	40	60	84	112
z	13	25	41	61	85	113

Notice that, by construction, y and z have no factors in common.

(iii) Consider the possibility that $z = y + 3$. Then

$$x^2 = (y+3)^2 - y^2$$
$$= 3(2y+3).$$

Hence x^2 is divisible by 3, so x must be divisible by 3. But then $x^2 = 3(2y+3)$ must be divisible by 9, so y must be divisible by 3. Further $z = y + 3$ must be divisible by 3, so we have shown that x, y and z all have a common factor 3. This contradicts the requirement that they have no factors in common. Hence there are no solutions of the type required in part (ii) with y and z differing by exactly 3.