YEARLY EXAMINATION

YEAR 9 2012

MATHEMATICS

Time Allowed – 85 minutes plus 5 minutes Reading time

INSTRUCTIONS:

- All questions may be attempted
- Start each section on a new page
- Write your name at the top of each page
- **Write** in **Pen** and draw diagrams in **Pencil**
- Department of Education approved calculators and templates are permitted
- Show all necessary working
- Marks may not be awarded for untidy or carelessly arranged work
- No grid paper is to be used unless provided with the examination paper
- **Teachers: Please collect each section separately.**

Section A (14 Marks)

Marks

1. Evaluate $\dfrac{1-\sin 30^0}{\sqrt{\pi+1}}$ to 2 decimal places. **1**

2. Find the value of x if $9^x = 27$. **1**

3. Simplify $\left(\dfrac{64}{125}\right)^{-\frac{2}{3}} = x^2$, find x. **1**

4. Simplify $\sqrt{27}+2\sqrt{3}-\sqrt{48}$. **2**

5. Solve for x: $(2x-5)^2 = 2x-5$. **2**

6. Make v the subject : $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$. **2**

7. Rationalize the denominator : $\dfrac{2}{3\sqrt{3}-4}$. **2**

8. A ball is thrown vertically upwards, with its height h, in metres, after a time of t seconds, being given by the formula $h = 8t - t^2$. Find the maximum height reached by the ball. **3**

Section B (14 Marks)

1. Factorise completely: $2x^3 - 54y^3$. **2**

2 Solve all the imaginary and real roots for : **4**
$(x^2+2)^2 - 4(x^2+2) + 3 = 0$

3. Draw a box and whisker diagram for the following set of data : **3**
65, 76, 83, 90, 62, 72, 74, 69, 80, 75, 65, 78.

4. a) Sketch $y = (x+3)(x-1)^2$. **2**

   b) Hence solve $(x+3)(x-1)^2 > 0$. **1**

5. Simplify $\dfrac{6^n - 2^n}{9^n - 3^n}$ **2**

Section C (15 Marks)

1. The formula $\tan\theta = \dfrac{m_1 - m_2}{1+m_1 m_2}$ allows you to find the angle θ between two straight lines where m_1, m_2 are the gradients of $y = 3x+5$ and $2x-3y+6=0$ respectively. Find the angle between these 2 lines to the nearest degree. **2**

Section C continued

Marks

2. Factorize completely: $2a - 3b + 4a^2 - 9b^2$. **2**

3.

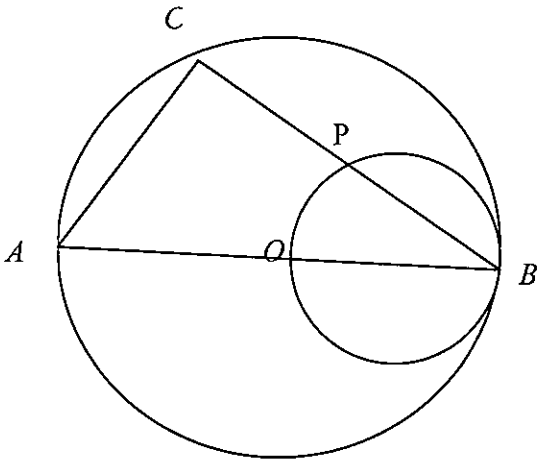


Diagram not to scale

In the diagram above, AB is the diameter of the circle ABC with centre O and OB is the diameter of the smaller circle OBP.

a) Prove that $OP \,//\, CA$. **2**

b) Prove $\Delta ACB \,///\, \Delta OPB$. **2**

c) Prove that the area of quadrilateral $AOPC$ is 3 times the area of ΔOPB. **2**

4. Two ships A and B leaves a port at the same time. A heads due west and B in a direction N 78° 26" W. After one hour B has traveled 20 km and observes ship A in a direction due south.

a) Draw the diagram indicating all the information above. **1**

b) How far is A from B? (answer to the nearest km) **2**

5. In Robby's class, the Mathematics test has a class average of 60 and standard deviation of 20 while English test has a class average of 60 and standard deviation of 10. **2**
If Robby scored 79 and 70 in his Mathematics and English tests respectively, in which test did he do better? Give reasons.

Section D (15 Marks)

Marks

1.

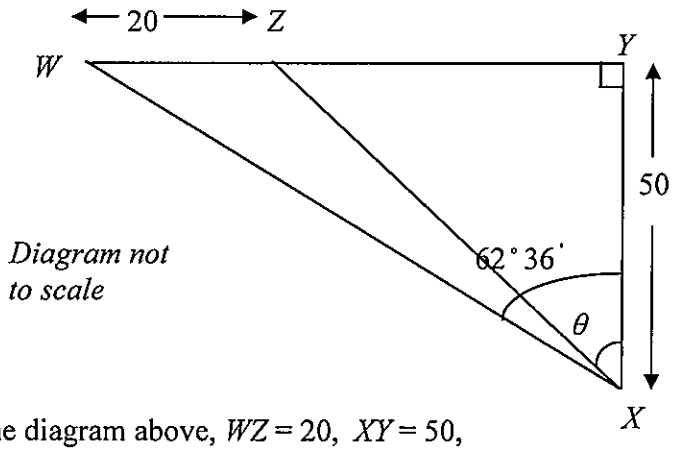


Diagram not to scale

3

In the diagram above, $WZ = 20$, $XY = 50$, $\angle WXY = 62^\circ 36'$ and $\angle ZXY = \theta$.
Copy the diagram onto your answer book and find the value of θ to the nearest minute.

2. Solve for x and y for the following simultaneous equations: 3

$$9^{x-2y} = 81$$
$$5^{x+y} = 1$$

3. Solve for x: $\dfrac{4x+3}{3-2x} + \dfrac{4+x}{2+3x} = \dfrac{x^2-18}{6x^2-5x-6}$. 3

4. Solve for the exact value(s) of x : $\sqrt{x+4} = x-3$ 3

5. 3

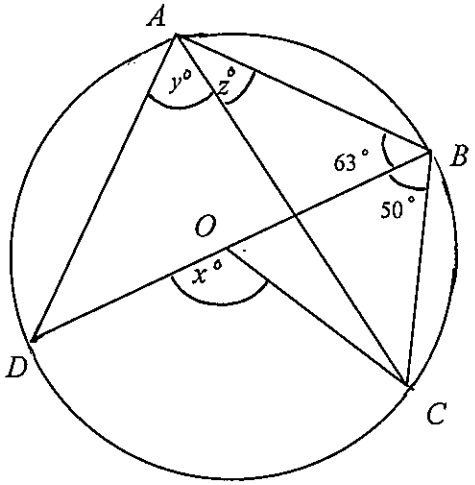


Diagram not to scale

O is the centre of the above cirlce. $\angle DBC = 50^\circ$ and $\angle ABD = 63^\circ$.
Copy the diagram onto your answer book.
Find the values of x, y and z, giving reasons.

Section E (15 Marks)

Marks

1. Mr Parker wanted to buy some lions and tigers for his private zoo. Lions cost \$2000 each while tigers cost \$5000 each. He has decided that he must have at least 5 but no more than 10 lions and he must have at least 2 tigers.
Mr Parker can afford to spend no more than \$ 40 000.

   Let the number of lions be x and the number of tigers be y.

   a) Write down all the constraints that govern Mr Parker's plan. **2**
   b) Graph the above constraints and shade in the feasible regions. **2**
   c) What is the greatest number of animals that can be bought? **1**
   d) If it costs \$50 per week to keep a lion and \$140 per week to keep a tiger, what is maximum cost to maintain the animals? **2**

2. Let $ABCD$ be a rhombus such that $\angle ABC = 130^\circ$. Let M and N be the mid-points of BC and AD respectively.
Draw the diagram showing all the above information.

   a) Prove that $MCNA$ is a parallelogram. **2**
   b) Prove that $\angle AMN + \angle ACN = 25^\circ$ **3**

3. A polynomial has a remainder of 3 and -2 when divided by $(x+1)$ and $(x-2)$ respectively. Find the remainder when the polynomial is divided by $x^2 - x - 2$. **3**

END OF PAPER

Year 7 Testing 2012

Section A

1) $0.24156\ldots = 0.25$ (2dp) # 1

2) $3^{2x} = 3^3 \quad \therefore x = 3/2$ # 1

3) $\left(\frac{125}{64}\right)^{\frac{2}{3}} = \left(\pm\frac{5}{4}\right)^2 \quad \therefore x = \pm\frac{5}{4}$ #1

4) $3\sqrt{3} + 2\sqrt{3} - 4\sqrt{3} = \sqrt{3}$ # 1+1

5) $2x-5 = 1$ or $2x-5 = 0$
$\therefore (2x-5)(2x-5-1) = 0$ 1
$x = 5/2$ or 3 # 1

6) $\frac{u-f}{fu} = \frac{1}{v}$ 1
$v = \frac{fu}{u-f}$ # 1

7) $\frac{2\times 3\sqrt{3}+4}{(3\sqrt{3}-4)(3\sqrt{3}+4)} = \frac{6\sqrt{3}+8}{27-16}$ 1
$= \frac{6\sqrt{3}+8}{11}$ # 1

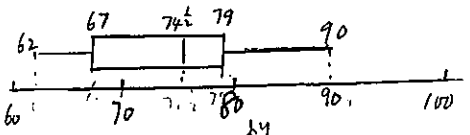
8) $h = -t(t-8)$
Axis of sym $t = \frac{-8}{-2} = 4$ 1
max height $(h) = -4(-4) = 16$ 1
Since h is concave down 1
$\therefore$ max ht = 16 m #

Section B

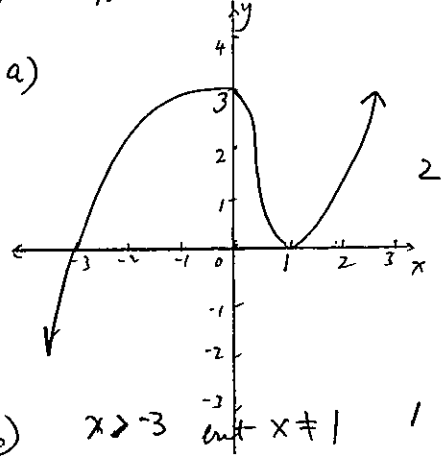
1) $2(x^3 - 27y^3)$
$= 2(x^3 - (3y)^3)$ 1
$= 2(x-3y)(x^2 + 3xy + 9y^2)$ #1

2) Let $x^2+2 = u$
$u^2 - 4u + 3 = 0$ 1
$(u-3)(u-1) = 0$ 1
$\therefore u = 1$ or 3 1
$x^2+2 = 1 \quad x^2 = -1 \quad x = \pm i$
or $x^2+2 = 3 \quad x^2 = 1 \quad x = \pm 1$
$\therefore x = \pm 1$ or ± 2 # 1

3) 62 65 69 72 74 71 76 78 80 81 90
CL = (65+69) ÷ 2 = 67 #
UL = 79
median = 74.5
3



4 a)



2

b) $x > -3$ but $x \neq 1$ 1

Section B

5) $\frac{3^n 2^{2n} - 2^n}{3^{2n} - 3^n} = \frac{2^n(3^n - 1)}{3^n(3^n - 1)}$ 1
$= \left(\frac{2}{3}\right)^n$ or $\frac{2^n}{3^n}$ # 1

Section C

1) $m_1 = 3 \quad m_2 = \frac{2}{3}$ 1
$\tan\theta = \frac{3 - \frac{2}{3}}{1+2} = \frac{2\frac{1}{3}}{3} = \frac{7}{9}$ # 1
$\theta = 37.87\ldots = 38°$ (nearest degree)

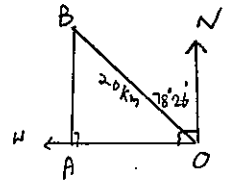
2) $2a - 3b + (2a-3b)(2a+3b)$ 1
$= (2a-3b)(1 + 2a + 3b)$ # 1

3) i) $\angle OPA = 90°$ (angle of semi-circle $= 90°$) $\frac{1}{2}$
Similarly $\angle ACB = 90°$ $\frac{1}{2}$
$\therefore OP \parallel CA$ (corresponding angles equal) 1

ii) $\angle OPA = \angle ACB = 90°$ $\frac{1}{2}$ (proved in i)
$\angle B$ is common $\frac{1}{2}$
$\therefore \triangle ACB \parallel \triangle OPB$ (equiangular) 1

iii) $\frac{\text{Area } \triangle OPB}{\text{Area } \triangle ACB} = \left(\frac{r}{d}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ 1
r = radius, d = diameter
$\therefore$ Area $AOPC$ = 3 times area of $\triangle OPB$.
$\frac{\text{Area } AOPC}{\text{Area } \triangle ACB} = \frac{3}{4}$ 1

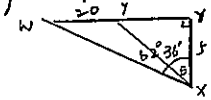
4 a)



1m
1

4b) $AO = 20\cos 78°26' = 4.010$ km
$= 4$ km (nearest)

5) $\overline{x}_M = 60 \quad \sigma_M = 20 \quad 79 = 60 + \frac{19}{20}\sigma$
$\overline{x}_E = 60 \quad \sigma_E = 10 \quad 70 = 60 + \frac{10}{10}\sigma$
$70 = 60 + \sigma$
$\therefore$ English better because it ranks at a higher percentile



Section D:

1) $\frac{WY}{50} = \tan 62°36'$, $\frac{WY-20}{50} = \tan$
$WY = 96.45\ldots97\ldots$
$= 96.4598$ 1
$\tan\theta = \frac{76.4598}{50} = 1.529195\ldots$ 1
$\theta = 62°35'59.58'' = 62°36'$ (nearest min)
1

2) $3^{2(x-2y)} = 3^4 \quad \therefore 2(x-2y) = 4$
$5^{(x+y)} = 5^0 \quad x+y = 0$
$3y = -2$
$y = -2/3$ # $\frac{1}{2}$
$\therefore x = \frac{2}{3}$ # $\frac{1}{2}$

3) $\frac{(4x+5)(2+5x)+(4+x)(1-2x)}{(3-2x)(2+3x)}$

LHS = $\frac{1}{2}$

$= \frac{8x+6+12x^2+9x+12-2x^2-5x}{6-4x+9x-6x^2}$ $\frac{1}{2}$

$= \frac{10x^2+12x+18}{-6x^2+5x+6}$ $\frac{1}{2}$ $= \frac{x^2-18}{6x^2-5x-6}$ (Rus)

$\therefore 10x^2+12x+18 = 18-x^2$

$11x^2+12x = 0$ $\frac{1}{2}$

$x(11x+12) = 0$ $\frac{1}{2}$

$\therefore x = 0$ or $\frac{-12}{11}$ # $\frac{1}{2}$

4) $x+4 = (x-3)^2 = x^2-6x+9$

$0 = x^2-7x+5$ 1

$x = \frac{7 \pm \sqrt{49-4(1)}}{2}$

$x = \frac{7 \pm \sqrt{29}}{2}$ 1

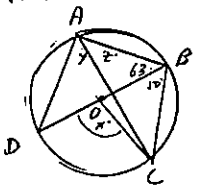
but $x \geq 3$ $\therefore x = \frac{7+\sqrt{29}}{2}$ only 1

5) $x = 50 \times 2 = 100°$ (angle at centre # twice angle at circumference) 1

$y = 50°$ (angle at circumference standing in same segment) 1

$z + y = 90°$ (angle in semi-circle = 90°)

$z = 90° - 50° = 40°$ # 1



Section C.

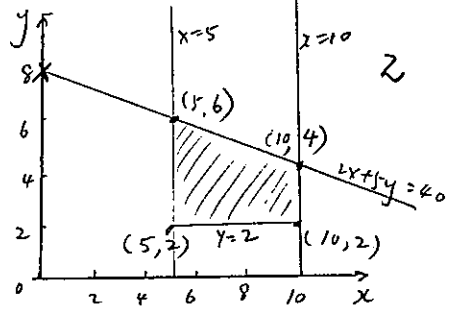
1a) $1 \leq x \leq 10$ $\frac{1}{2}$

$y \geq 2$ $\frac{1}{2}$

$2000x + 5000y \leq 40000$ 1

(ie $2x+5y \leq 40$)

b)



2

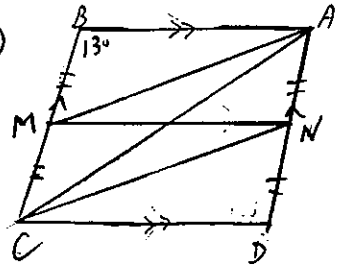
c) Greatest no of animals is 10 lions and 4 tigers. 1

d) At (5, 6) cost = $5 \times 50 + 6 \times 140$
$= \$1090$ $\frac{1}{2}$

At (10, 4) cost = $10 \times 50 + 4 \times 140 = \$1060$ $\frac{1}{2}$

For (5, 2), (10, 2) cost will be less than \$1060 ∴ next cost is \$1090 1

2)



To prove $\angle AMN + \angle NCD = 2s$

Proof:

a) $\frac{1}{2}$ BC = AD (opposite sides of rhombus are equal)

$\frac{1}{2}$ MC = AN (M, N are mid-points of equal sides BC, AD respectively)

$\frac{1}{2}$ BC ∥ AD (opposite sides of rhombus are parallel)

$\frac{1}{2}$ MCNA is a parm. (one pair of opposite sides equal & parallel)

b) $\frac{1}{2}$ Similarly MCDN is also a parm

$\frac{1}{2}$ MA ∥ CN (opposite sides of parm. MCNA are parallel)

$\frac{1}{2}$ $\angle AMN = \angle MNC$ (alternate angles equal, MA ∥ CN)

Similarly,

$\frac{1}{2}$ $\angle MNC = \angle NCD$

$\therefore \angle AMN = \angle NCD$

$\angle ABC = 130°$ (given)

$\frac{1}{2}$ $\angle BCD = 50°$ (adjacent angles of parm. are supplementary)

$\frac{1}{2}$ $\angle NCD + \angle ACN = 2s°$ (diagonal bisects vertex angle in rhombus)

$\angle AMN + \angle ACN = 2s°$ ($\angle AMN = \angle NCD$, proved before)

#

3) Degree of remainder < Degree of divisor

Let the remainder polynomial be $P(x) = ax + b$

$\therefore P(-1) = 3$, $P(2) = -2$

$\frac{1}{2}+\frac{1}{2}$ $-a + b = 3$

$2a + b = -2$

$3a = -5$ $\therefore a = -5/3$

$b = 3 - 5/3$ $b = \frac{4}{3}$

1 $\therefore$ Remainder is $\frac{-5x}{3} + \frac{4}{3}$ #

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Section C

1. $y = 3x+5 \Rightarrow m_1 = 3$

$2x-3y+6=0 \Rightarrow y = \frac{2}{3}x+2$

$\therefore m_2 = \frac{2}{3}$

$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$= \left|\frac{3-\frac{2}{3}}{1+3\times\frac{2}{3}}\right|$

$= \left|\frac{9-2}{3+6}\right|$

$= \frac{7}{9}$

$\theta = 37.87...^\circ$

$= 38^\circ$ to nearest degree

($\frac{1}{2}$) for m_1

($\frac{1}{2}$) for m_2.

$\frac{1}{2}$ for correct substitution + obtaining $\frac{7}{9}$.

($\frac{1}{2}$) Correct Answer

2. $(2a-3b)+4a^2-9b^2$

$(2a-3b)+(2a-3b)(2a+3b)$

$(2a-3b)(1+2a+3b)$

1 mark

1 mark Correct Answer

3. Join OP

$\angle OPB = \angle ACB = 90^\circ$ (Angle in a semi-circle = 90°)

$\therefore OP \parallel CA$ (Corresponding Angles are equal)

In $\triangle ACB$ and $\triangle OPB$

1. $\angle B$ is common
2. $\angle ACB = \angle OPB$ (Proven above)

$\therefore \triangle ACB \sim \triangle OPB$ (equiangular)

$\frac{1}{2}$ $\angle OPB = 90^\circ$

$\frac{1}{2}$ $\angle ACB = 90^\circ$

Must give reason for at least one $\angle = 90$

$\frac{1}{2}$ with reason

$\frac{1}{2}$ with reason

1 Must have equiangular

No AAA

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(iii) $\frac{\text{Area } \triangle OPB}{\text{Area } \triangle ACB} = \left\{\frac{OB}{AB}\right\}^2$

$= \left\{\frac{6}{12}\right\}^2$

$= \left(\frac{1}{2}\right)^2$

$= \frac{1}{4}$

$= \frac{2 \text{ unit}^2}{8 \text{ unit}^2}$

1 mark for ratio $= \frac{1}{4}$

Area AOPC = Area ACB − Area OPB

$= (4a-a)$ unit^2

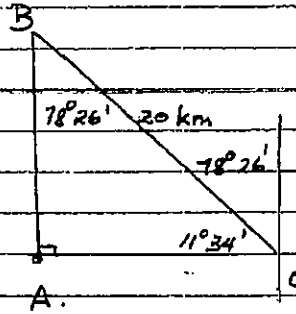
$= 3a$ unit^2.

$\therefore \frac{\text{Area AOPC}}{\text{Area OPB}} = \frac{3a}{a}$

$= 3:1$

1 mark $\left(\begin{array}{c}4-1\\=3\\\therefore 3:1\end{array}\right)$

No a units, 3a units necessary



$\cos 78^\circ 26' = \frac{AB}{20}$

$AB = 20 \cos 78^\circ 26'$

$= 4.010...$ km

$= 4$ km (nearest km)

(1) Diagram Must have right angled $\triangle$ and correctly placed angle of either $78^\circ 26'$ OR $11^\circ 34'$

1 Correct statement Using Trigonometry May have used $AB = \sin 11^\circ 34'$

Marks incorrect diagram than AB correct i.e. C.F.P.A

1 Correct Answer.

Award 1½

5

English $\bar{x}+1\sigma = 60+10 = 70$

Mathematics $\bar{x}+1\sigma = 60+20 = 80$

English is better 1 σ above mean

$\frac{1}{2}$ OR Similar Statement

$\frac{1}{2}$ OR Similar Statement

1 Correct Decision with reasoning.