Name:	
Class:	

YEARLY EXAMINATION

YEAR 9 2013

MATHEMATICS

Time Allowed – 85 minutes plus 5 minutes Reading time.

INSTRUCTIONS:

- Start each section on a new page
- **Write your Name and Class at the top of each page**
- **Write** in **Pen** and draw diagrams in **Pencil**
- Department of Education approved calculators are permitted
- The use of mathematical templates are permitted.
- Show all necessary working
- Marks may not be awarded for untidy or carelessly arranged work
- No grid paper is to be used unless provided with the examination paper
- **Teachers: Please collect each section separately.**

Outcome	A	A	B	B	C	C	D	D	E	E	Total
Number			1	/4					2	/3	/7
Measure			5	/2			5,6	/5			/7
Algebra	1,2,3,4	/14	2,3	/5	1,3	/11	1,2,3,4	/9	1,3,4,5,6	/11	/50
Geometry			4	/4	2	/4					/8
Statistics	5	/1									/1
Total		/15		/15		/15		/14		/14	/73

Section A (15 marks)

Marks

1. Simplify: (a) $6v^0$ **1**

   (b) $(3w^4)^{\frac{1}{2}}$ **1**

   (c) $\sqrt{48} - \sqrt{75}$ **1**

2. Simplify fully: $8(2x^2 - x)^3 \div [2(2x^2 - x)^2(2x - 1)]$ **2**

3. Solve: (a) $7^{3x-1} = 7$ **2**

   (b) $5^{x+2} + 5^{x+1} = 3750$ **2**

4. A shop sells packets of lollies and ice-blocks every week. To make a profit the shopkeeper must sell at least 500 packets of lollies. Unfortunately the shopkeeper only has a small store and so therefore can only stock a maximum of 1300 packets of lollies and 1400 ice-blocks. He sells at least twice as many packets of lollies as ice-blocks.
   Let x be the number of packets of lollies sold and y the number of ice-blocks sold.

   (a) Write these constraints as inequations. **1**
   (b) Neatly graph the above constraints. **1**
   (c) Write down the coordinates of the four vertices. **2**
   (d) If the profit can be expressed as P = 0.5x + 0.1y in cents, find which vertice will produce the maximum profit in dollars for the shopkeeper. **1**

5. Mr Woo told his Year 9 Maths class that the test results were positive skewed, explain clearly what Mr Woo meant. **1**

Section B (15 marks) Start a new page

1. Two dice are rolled simultaneously; find the probability of obtaining a total:

(a) of six. **1**

(b) greater than nine. **1**

(c) of five if at least one of the dice shows an even number. **2**

2. Solve for a and b if $2a - 3b = 9$ and $4a + 3b = 24$. **2**

3. 1250 adults and children went to the 11am movie session. The cost of a children's ticket is \$11 and an adult's ticket is \$16. Form a pair of simultaneous equations and solve them to find the number of adults and children if the takings for this session were \$18 225. **3**

4. WXYZ is a kite, <XYZ is 46^0 and <ZWX is 102^0, draw a diagram to illustrate this information and find the size of angle X giving full reasons for your answer. **4**

5. In ΔMNO, angle N is 90^0, MN = 23.1cm and NO = 31.6cm. Find the size of angle MON to the nearest minute. **2**

Section C (15 marks) Start a new page

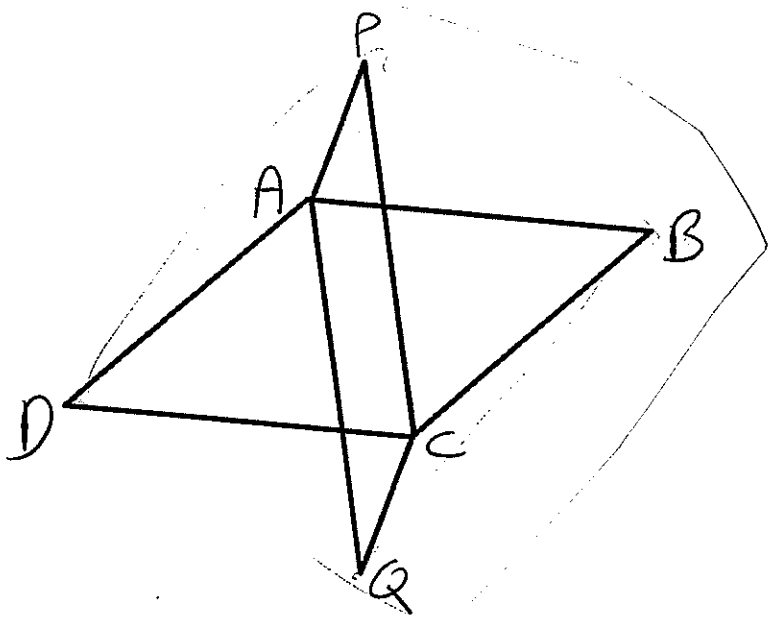
1. Solve for x:

   (a) $2x = 6x^2$ **2**

   (b) $2x^2 - 13x + 20 = 0$. **3**

   (c) $ax - b = a + bx$ **1**

2. In the diagram below ABCD and APCQ are parallelograms. Copy the diagram neatly onto your page and prove that PBQD is a parallelogram. **4**



3. A soccer ball is kicked and its trajectory forms a parabolic arc. The height is given by

h = $(27t - \frac{1}{3}t^2)$, where t is the time in seconds and h is the height ir centimetres.

(a) Find the height of the soccer ball when t = 21s. **1**

(b) When does the soccer ball return to the ground? **2**

(c) Find the maximum height of the soccer ball, giving reasons for your answer. **2**

Section D (14 marks) Start a new page

1. (a) Neatly sketch $y = 2x - 4$ on a number plane clearly showing both intercepts. **1**

(b) On the same number plane sketch the region where $y - 2 \leq 0$ and $2x - y - 4 \leq 0$ hold simultaneously. **2**

2. The midpoint of (x, 13) and (-3, 7) is (1, 10). Find the value of x. **1**

3. Find the equation of the line in general form if it passes through (4, -1) and has a gradient of $\frac{-1}{2}$. **2**

4. Solve for x: $x + 3 = 2 + \frac{5}{x-3}$. **3**

5. Evaluate correct to four decimal places: **1**

$$\sin 56^\circ 13' \cos 11^\circ 13' - \cos 56^\circ 13' \sin 11^\circ 13'$$

6. Ms Jenns takes a group of hikers on a hike; they walk 14.5km due North from Point B to Point A. Unfortunately Scott was playing on his iphone and walked $040°T$ fro n Point B. The group decide to stop at Point A and wait for Scott.

(a) How far would Scott have to walk in order to be due East of Ms Jenns and the hikers when they are at Point A? **2**

(b) Scott walks xkm on this bearing then he receives a text telling him to turn and walk in a straight line on a bearing of $310°T$ to the Point A and he can meet the group there. Find the distance x correct to one decimal place. **2**

Section E (14 marks) Start a new page

1. $3x - 4y + 7 = 0$ is perpendicular to $ax + 3y + 10 = 0$. Find the value of a, giving reasons for your answer. **1**

2. A dice is biased so that the probability of rolling a four is 50%, all other numbers have an equal chance of occurring. This dice is rolled twice, find the probability of rolling:

(a) a double four. **1**

(b) A double four if it is known that at least one of the die show a four. **2**

3. Given P(x) = $3x^4 + x^2 - x + 5$ and Q(x) = $x - 2$, find:

(a) P(x).Q(x) **1**

(b) P(x) ÷ Q(x); express your answer in the form P(x) = Q(x).D(x) + R **2**

(c) the degree of $[P(x)]^2$ **1**

4. The remainder when p(x) is divided by $(x + 2)$ is -3, and when p(x) is divided by $(x - 2)$ the remainder is 6. Find the remainder when p(x) is divided by $x^2 - 4$. **2**

5. Neatly sketch y = $x(x - 1)^3(x + 2)$ on a number plane clearly showing all intercepts. **2**

6. (a) If $(x-1)$ is a factor of $ax^3 + (b - a)x^2 + (c - b)x - c$, find the other factor. **1**

(b) State the condition on a, b and c for the cubic expression to have three real roots. **1**

END OF PAPER

Yr 9 Yrly 2013

A

1(a) $6v^0 = 6$ (1)

(b) $(3w^4)^{1/2} = \sqrt{3}w^2$ (1)

(c) $\sqrt{48} - \sqrt{75} = 4\sqrt{3} - 5\sqrt{3}$ (1½)

$= -\sqrt{3}$

2. $$\frac{8(2x^2-x)^3}{2(2x^2-x)^2(2x-1)}$$

$$= \frac{4(2x^2-x)}{2x-1}$$ (1)

$$= \frac{4x(2x-1)}{2x-1}$$

$= 4x$ (1)

3. (a) $7^{3x-1} = 7^1$ (1)

$3x = 2$

$x = 2/3$ (1)

(b) $5^{x+1}(5+1) = 3750$

$5^{x+1} = 625$ (1)

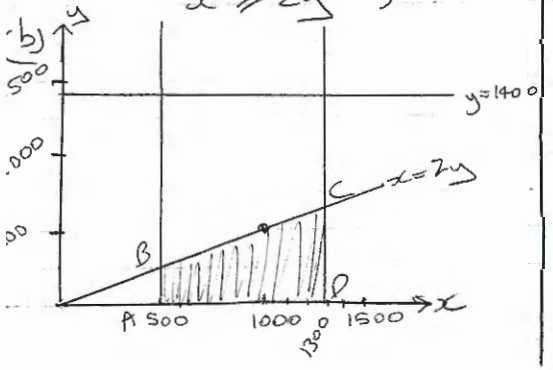
$5^{x+1} = 5^4$

$\therefore x = 3$ (1)

4. (a) $x \geq 500$, $x \leq 1300$ } (1½)

$y \leq 1400$, $x \geq 2y$ } (1½)

(b)



(c) A (500, 0) C (1300, 650)

B (500, 250) D (1300, 0)

(d) $P = 0.5x + 0.1y$

C will produce max. profit

$\therefore P = 0.5(1300) + 0.1 \times 650$

$= 715$

$\therefore P = \$7.15$ (1)

5. There are more results to the right of the mean, the median will be to the right of the mean. (1)

SECTION B

1(a) $P(\text{total of six}) = \frac{5}{36}$ (1)

(b) $P(\text{total} > 9) = \frac{6}{36} = \frac{1}{6}$ (1)

(c) $P(\text{total of 5 ...}) = \frac{4}{27}$ (2)

2. $2a - 3b = 9$ ... (1)

$4a + 3b = 24$ ... (2)

(1) + (2)

$6a = 33$

$a = 5\frac{1}{2}$ (1)

sub into (1)

$11 - 3b = 9$

$-3b = -2$

$b = 2/3$ (1)

3. let a = no. of adults and c = no. of children

$a + c = 1250$ ... (1)

$16a + 11c = 18225$ ... (2)

(1) × 16

$16a + 16c = 20000$ ... (3)

(3) − (2)

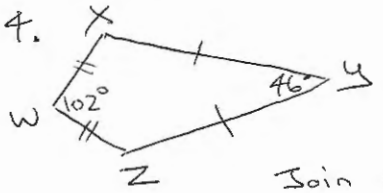
$\therefore 5c = 1775$

$c = 355$ (3 mk)

$\therefore 895 = a$

There were 895 adults and 355 children at the cinema.

4.



(4 mk)

Join W to Y

In $\triangle$s WXY, WZY

ZY = XY (adjacent sides of a kite are equal.)

similarly ZW = WX

WY is common

$\therefore \triangle WXY \equiv \triangle WZY$ (SSS)

$\therefore \angle WXY = \angle WZY$ (corresponding angles in congruent triangles are equal).

$2x\,W\hat{X}Y + 102° + 46° = 360°$

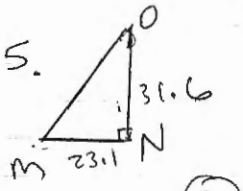
(angle sum of a quadrilateral is 360°).

$2x\,W\hat{X}Y + 148° = 360°$

$2x\,W\hat{X}Y = 212°$

$W\hat{X}Y = 106°$

5.



$\tan\theta = \frac{23.1}{31.6}$

$\tan\theta = 0.73101$

$\theta = 36°10'$ (nearest minute)

(2)

SECTION C

1.(a) $2x = 6x^2$

$0 = 6x^2 - 2x$

$0 = 2x(3x - 1)$ (1)

$x = 0$ or $x = 1/3$ (1)

(b) $2x^2 - 13x + 20 = 0$

$2x^2 - 8x - 5x + 20 = 0$

$2x(x-4) - 5(x-4) = 0$

$(2x-5)(x-4) = 0$ (1)

$x = 5/2$ or $x = 4$ (2)

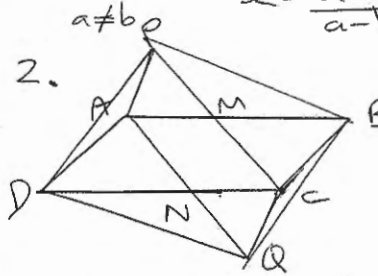
(c) $ax - b = a + bx$

$ax - bx = a + b$

$x(a-b) = a+b$

$x = \frac{a+b}{a-b}$ (1)

2. $a \neq b$



Join P to B, B to Q, Q to D and D to P.

PC meets AB at M.

AQ meets DC at N.

In AMCN

AM ∥ NC (segments of parallel lines AB & DC)

Similarly AN ∥ MC

$\therefore$ AMCN is a parm (both pairs of opposite sides parallel)

In $\triangle$PCD, $\triangle$QAB

PC = QA (opposite sides of parm PCQA are equal.)

similarly $CD = AB$

$\angle DCP = \angle QAB$ (opposite angles of parm AMCN are equal).

$\therefore \triangle PCD \equiv \triangle QAB$ (SAS).

$\therefore DP = BQ$ (corresponding angles of congruent triangles are equal).

now $B\hat{C}D = D\hat{A}B$ (opposite angles of parm ABCD are equal).

$\therefore B\hat{C}P + \angle PCD = B\hat{A}Q + Q\hat{A}D$ (sum of adjacent angles)

$B\hat{C}P = Q\hat{A}D$ (proven above)

$\therefore PCD = B\hat{A}Q$ (by subtraction)

In $\triangle$'s PBC, DAQ

$DA = BC$ (opposite sides of parm ABCD are equal).

similarly $AQ = PC$

$PCD = BAQ$ (proven above)

$\therefore \triangle PBC \equiv \triangle DAQ$ (SAS)

$\therefore PB = DQ$ (corresponding sides in congruent triangles are equal.)

$\therefore PBQD$ is a parallelogram (2 pairs of opposite sides are equal).

3.(a) $h = 27t - \frac{1}{3}t^2$

$t = 21, h = ?$

$h = 27 \times 21 - \frac{1}{3} \times 21^2$

$= 567 - 147$

$\therefore h = 420$ (1)

$\therefore$ The height is 420cm.

(b) when $h = 0$ $t = ?$

$\therefore 0 = 27t - \frac{1}{3}t^2$

$0 = 81t - t^2$

$0 = t(81 - t)$ (1)

$\therefore t = 0$ or $t = 81$ (1)

returns after 81 seconds

(c) axis of symmetry is $-\frac{b}{2a}$ (1/2)

$= 40\frac{1}{2}$ s.

sub $t = 40\frac{1}{2}$ into "h" to get the max. height as the maximum occurs at the vertex on a concave down parabola (1/2) (Or // using a graph to justify where the maximum occurs.).

$\therefore h = 27 \times 40\frac{1}{2} - \frac{1}{3} \times 40\frac{1}{2}^2$

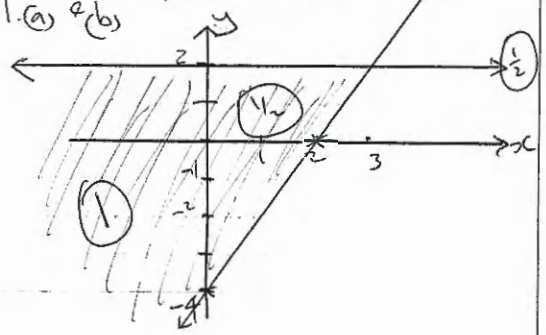
$= 1093.5 - 546.75$ (1/2)

$= 546.75$

$\therefore$ maximum height is 546.75 cm (1/2)

SECTION D.

1.(a) & (b)



Q2. $\frac{x+-3}{2} = 1$

$x - 3 = 2$

$x = 5$ (1)

Q3. $y - y_1 = m(x - x_1)$

$y + 1 = -\frac{1}{2}(x - 4)$ (1)

$-2y - 2 = x - 4$

$0 = x + 2y - 2$ (1)

Q4. $x + 3 = 2 + \frac{5}{x-3}$ $x \neq 3$

$(x+3)(x-3) = 2(x-3) + 5$ (1)

$x^2 - 9 = 2x - 6 + 5$

$x^2 - 2x - 8 = 0$ (1)

$(x-4)(x+2) = 0$ (1)

$x = 4$ or -2

* $(x - 4\sqrt{14}$ if they forget to multiply the 2 by $(x-3)$

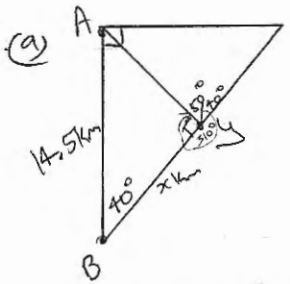
5. $\sin 56°13' \cos 11°13' - \cos 56°13' \sin 11°13'$

$= 0.815270 - 0.108163426$

$= 0.707106781$ (1/2)

$= 0.7071$ (4dp) (1/2)

6.(a)



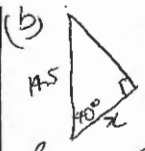
$\cos 40° = \frac{14.5}{y}$

$y = \frac{14.5}{\cos 40°}$

$y = 18.9284057$

$y = 18.9$ km

(b)



(1/2)

$\cos 40° = \frac{x}{14.5}$ (1/2)

$x = 14.5 \times \cos 40°$ (1/2)

$x = 11.10764443$

$x = 11.1$ km (1dp) (1/2)

SECTION E

1. $3x - 4y + 7 = 0$ $\therefore m_1 = 3/4$

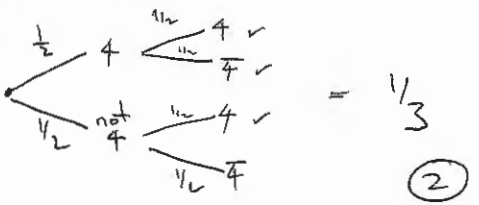
$ax + 3y + 10 = 0$ $\therefore m_2 = -a/3$

(1) $\therefore \frac{3}{4} \times \frac{-a}{3} = -1$ (product of perpendicular lines is -1).

$\therefore a = 4$

2.(a) P(double 4) $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ (1)

(b) P(double 4 if ...).



(2)

3(a) $P(x) \cdot Q(x)$

$= (3x^4 + x^2 - x + 5)(x - 2)$

$= 3x^5 + x^3 - x^2 + 5x - 6x^4 - 2x^2 + 2x - 10$ (1)

$= 3x^5 - 6x^4 + x^3 - 3x^2 + 7x - 10$

(b)

$$x-2 \overline{)\,3x^4 + 0x^3 + x^2 - x + 5}$$

$3x^3 + 6x^2 + 13x + 25$

$3x^4 - 6x^3$

$6x^3 + x^2$ (1)

$6x^3 - 12x^2$

$13x^2 - x$

$13x^2 - 26x$ (1/2)

$25x + 5$

$25x - 50$

$\therefore P(x) = (x-2)(3x^2+6x^2+13x+25) + 55$ (½)

9) degree of $[P(x)]^2 = 8$ (1)

Q4.

$P(x) = (x-2)(x+2)Q(x) + ax + b$

$P(-2) = -2a + b = -3 \quad \ldots (1)$ (½)

$P(2) = 2a + b = 6 \quad \ldots (2)$

(2) − (1)

$4a = 9$

$a = 9/4$ (½)

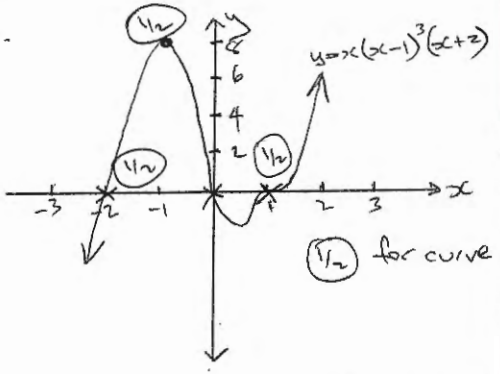
sub into (2)

$\frac{9}{2} + b = 6$ (½)

$\therefore b = 1\frac{1}{2}$

$\therefore$ Remainder is $\frac{9}{4}x + \frac{3}{2}$ (½)

5.



(½) (½) (½) (½) for curve

2(a)

$$x-1 \;\overline{)}\; ax^3 + (b-a)x^2 + (c-b)x + c$$

$ax^3 - ax^2$

$bx^2 + cx - bx$

$bx^2 - bx$

$cx - c$

$cx - c$

(½) (½)

$\therefore$ other factor is (ax^2+bx+c)

Q6(b) For 3 different roots, then ax^2+bx+c needs to have 2 roots, so the discriminant (½) needs to be greater than zero.

ie $b^2 - 4ac > 0$

$b^2 > 4ac$ (½)