Name:	
Class:	

YEARLY EXAMINATION

YEAR 9 2014

MATHEMATICS

Time Allowed – 85 minutes plus 5 minutes Reading time.

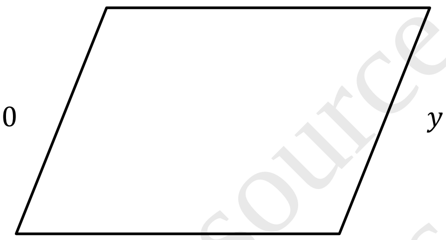
INSTRUCTIONS:

- Start each section on a new page
- **Write your Name and Class at the top of each page**
- **Write** in **Pen** and draw diagrams in **Pencil**
- Department of Education approved calculators are permitted
- The use of mathematical templates are permitted.
- Show all necessary working
- Marks may not be awarded for untidy or carelessly arranged work
- No grid paper is to be used unless provided with the examination paper
- **Teachers: Please collect each section separately.**

Outcome	A	A	B	B	C	C	D	D	E	E	Total
Financial	1	/4	1.2	/6							/10
Algebra	2,3	/5			1,2	/6	1,2	/15	1	/3	/29
Trigonometry			3d	/2	3,4	/8			3,4,5	/8	/18
Co-ord geom			3a,b,c	/6							/6
Geometry	4,5	/5							2	/4	/9
Total		/14		/14		/14		/15		/15	/72

Section A (14 marks) **(START A NEW PAGE)**

1. Ray has the option of choosing between two types of investment. Plan A offers simple interest at 3% per quarter for 3 years, whereas Plan B offers compound interest of 11% p.a. for 3 years. Ray has $2000 to invest.
   a) Find the interest earned on Ray's investment if he chooses Plan A. **(2)**
   b) Compare and determine which plan will give the better return. **(2)**
2. By using the graphical method, solve the following simultaneous equations: $x + 2y = 7$ and $2x - y = -1$. **(2)**
3. Dr. Wong has $20000 to invest. He invests a part at 6% p.a. simple interest and the rest at 7% p.a. simple interest. After one year, he earns $1280 in interest. How much did he invest at each rate? **(3)**
4. State two tests that can be used to prove that a quadrilateral is a parallelogram. **(2)**
5. Given that the following shape is a parallelogram, find the perimeter of this parallelogram (to the nearest whole number) providing reasons. **(3)**



Section B (14 marks) **(START A NEW PAGE)**

1. The value of a new car depreciates at 5% p.a. If a new car was bought for $15700, what will its value be after 4 years? **(2)**
2. Molly earns $64500 each financial year. Her deductible allowance is $3497. The following table shows the tax rate for 2014-2015. There is also a Medicare levy of 2%.

Taxable income	**Tax on this income**
0 – $18,200	Nil
$18,201 – $37,000	19c for each $1 over $18,200
$37,001 – $80,000	$3,572 plus 32.5c for each $1 over $37,000
$80,001 – $180,000	$17,547 plus 37c for each $1 over $80,000
$180,001 and over	$54,547 plus 45c for each $1 over $180,000

a) What is Molly's total taxable income? **(1)**

b) Molly has already paid \$17900 in tax. During this financial year, would she have to pay more tax or receive a tax rebate (Medicare levy is included in the tax)? How much would it be? **(3)**

3. The point A(-7,0), B(-9,3) and C(0,9) are the vertices of a triangle.

   a) Show that ΔABC is a right-angled triangle. **(2)**

   b) Find the length of AB. **(2)**

   c) Find the area of ΔABC. **(2)**

   d) Calculate the size of ∠ACB to the nearest degree. **(2)**

Section C (14 marks) **(START A NEW PAGE)**

1. Solve for x:

   a) $2^x = \frac{1}{4}$ **(1)**

   b) $9^{2x-1} = 27^2$ **(2)**

2. When $x^3 + px^2 + p^2x - 36$ is divided by $x - 3$ the remainder is 21. Find the possible values of p. **(3)**

3. From the top of a 200 metres high building, the angle of depression to the bottom of a second building is 20°. From the same point, the angle of elevation to the top of the second building is 10°. Calculate the height of the second building in metres to 2 decimal places. **(3)**

4. Ship A leaves the dock and sails on a bearing of 028°T at a speed of 24km/h. Ship B leaves the dock at the same time on a bearing of 118°T at a speed of 28km/h. They travel for 4 hours.

   a) Sketch a diagram showing all the information given. **(1)**

   b) How far apart are the two ships? (to the nearest kilometre) **(2)**

   c) What is the bearing of Ship A from Ship B (providing reason)? **(2)**

Section D (15 marks) **(START A NEW PAGE)**

1. If $P(x) = x^3 + 2x^2 - 5x - 6$ and $A(x) = x + 2$,

   a) Find the degree and the leading coefficient of $P(x)$. **(1)**

   b) Find $P(x) \times A(x)$. **(1)**

   c) Find $P(x) \div A(x)$, writing your answer in the form $P(x) = A(x)Q(x) + R(x)$. **(2)**

   d) Factorise $P(x)$ fully. **(3)**

   e) Neatly sketch the graph of $P(x)$ showing all essential features. **(3)**

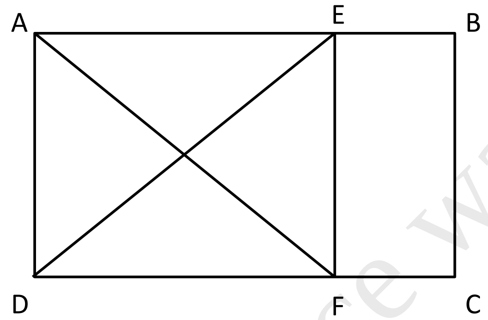
   f) Using your graph, solve the inequality $x^3 + 2x^2 - 5x - 6 > 0$. **(1)**

2. A cannon ball is fired from a hill which is 80 metres above a lake. The cannon ball's height (h) above the surface of the lake is given by $h = -16t^2 + 64t + 80$, where t is the time in seconds and h is measured in metres.

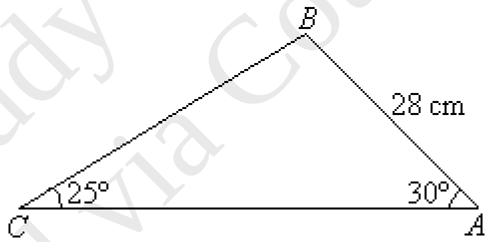
a) After how many seconds will the cannon ball hit the surface of the lake? **(2)**
b) What is the maximum height reached by the cannon ball? **(2)**

Section E (15 marks) **(START A NEW PAGE)**

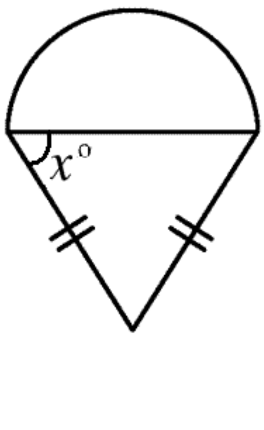
1. If A and B are the remainders when the polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $x + 1$ and $x - 2$ respectively and if $2A + B = 6$ find the value of a. **(3)**
2. Given that ABCD is a rectangle and that AF=DE. Prove that AEFD is a rectangle. **(4)**



3. In a right angled triangle, $\tan x = \frac{3}{4}$. Find $\sin x$ and $\cos x$. **(3)**
4. Find the length of BC by first splitting the triangle into two smaller right-angled triangles. Write your answer correct to 2 decimal places. **(2)**



5. The diagram shows a semi-circle and an isosceles triangle. The areas of the two shapes are equal. Find the value of $\tan x°$, leaving your answer in exact form. **(3)**



YR 9 YEARLY 2014

Section A

1. a) Plan A:

$I = PRN$ (1)

$I = 2000 \times 0.03 \times (4 \times 3)$

$= \$720$ (1)

b) Plan B:

$A = P(1+r)^n$

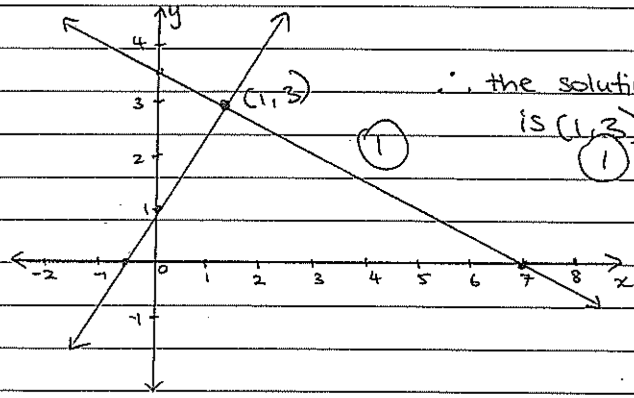
$= 2000(1+0.11)^3$

$= \$2735.26$ (1)

$I = 2735.26 - 2000$

$= \$735.26$

$\therefore$ Plan B will have the better return. (1)



$\therefore$ the solution is (1,3) (1) (1)

3. let the part invested at 6% be x (1)

let the part invested at 7% be y.

$x + y = 20\,000 \Rightarrow x = 20000 - y$ —(1) (1)

$0.06x + 0.07y = 1280$ —(2)

Sub. (1) into (2)

$0.06(20000 - y) + 0.07y = 1280$

$1200 - 0.06y + 0.07y = 1280$

$0.01y = 80$

$y = 8000$

when $y = 8000$, $x = 20000 - 8000$

$= 12000$

$\therefore$ He invested \$12000 at 6% p.a. and \$8000 at 7% p.a. (1)

4. - Both pairs of opposite sides parallel
- Both pairs of opposite sides equal
- Both pairs of opposite angles equal
- Diagonals bisect each other
- A pair of sides are equal and parallel

Any 2. (2)

5. $x + 30 = 2x - 10$ (opposite sides of a parallelogram are equal)

$x = 40$

$y + 10 = 2y - 10$ (opposite sides of a parallelogram are equal)

$y = 20$

$\therefore$ Perimeter $= 2(20) - 10 + 20 + 10 + 40 + 30 + 2(40) - 10$

$= 200$ units.

Section B

1. $A = P(1-r)^n$ 1

$= 15700(1-0.05)^4$

$= \$12787.75$ 1

2 a) Taxable income:

$\$64500 - 3497 = \$61003$ 1

b) Tax $= 3572 + (61003 - 37000) \times 0.325$

$= \$11372.98$

medicare $= 61003 \times 0.02$ 1

$= \$1220.06$

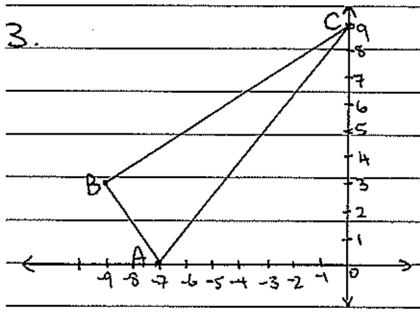
Tax due $= \$11372.98 + \$1220.06$

$= \$12593.04$

$\therefore 17900 - 12593.04 = \$5306.96$

$\therefore$ she would receive refund of \$5306.96 1

3.



a) $m_{AB} = \dfrac{0-3}{-7+9}$ $\quad m_{BC} = \dfrac{9-3}{0+9}$

$= \dfrac{-3}{2}$ $\quad = \dfrac{6}{9}$

$= \dfrac{2}{3}$ 1

$m_{AB} \times m_{BC} = -\dfrac{3}{2} \times \dfrac{2}{3}$

$= -1$

$\therefore AB \perp BC$ 1

$\therefore \triangle ABC$ is a right-angled triangle ($AB \perp BC$)

b) $AB = \sqrt{(-9+7)^2 + (3-0)^2}$ 1

$= \sqrt{(-2)^2 + 3^2}$

$= \sqrt{13}$ 1

c) $BC = \sqrt{(0+9)^2 + (9-3)^2}$ 1

$= \sqrt{117}$

$\therefore$ Area of $\triangle ABC = \frac{1}{2} \times \sqrt{13} \times \sqrt{117}$ 1

$= 19.5 \text{ units}^2$

d) $\tan \angle ACB = \dfrac{\sqrt{13}}{\sqrt{117}}$ 1

$\angle ACB = \tan^{-1}\left(\dfrac{\sqrt{13}}{\sqrt{117}}\right)$

$= 18.43\ldots$

$= 18^\circ$ (nearest degree) 1

Section C

1. a) $2^x = 4^{-1}$

$2^x = 2^{-2}$ (1)

$\therefore x = -2$

b) $3^{2(2x-1)} = 3^{3\times 2}$ (1)

$\therefore 2(2x-1) = 6$

$2x - 1 = 3$

$2x = 4$

$\therefore x = 2$ (1)

2. let $P(x) = x^3 + px^2 + p^2x - 36$

$P(3) = 3^3 + p(3)^2 + p^2(3) - 36$ (1)

$= 3p^2 + 9p - 9$

$P(3) = 21$

$\therefore 3p^2 + 9p - 9 = 21$ (1)

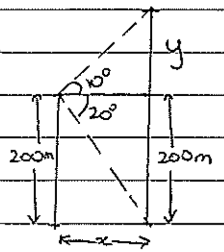
$3p^2 + 9p - 30 = 0$

$p^2 + 3p - 10 = 0$

$(p+5)(p-2) = 0$ (1)

$\therefore p = -5$ or 2

3.



$\tan 20° = \dfrac{200}{x}$

$x = \dfrac{200}{\tan 20}$ (1)

$\tan 10 = \dfrac{y}{x}$

$y = x\tan 10$

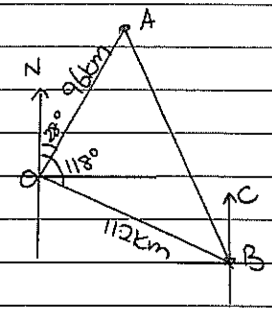
$= \dfrac{200\tan 10}{\tan 20}$ (1)

$\therefore$ height of second tower is:

$200 + y = 200 + \dfrac{200\tan 10}{\tan 20}$

$= 296.89\text{m}$ (2dp.) (1)

4. a)



(1)

b) $\angle BON - \angle AON = 90°$ (1)

$\therefore AB^2 = 96^2 + 112^2$ (by pythagoras theorem)

$AB = \pm\sqrt{21760}$

$= 147.512\ldots$, since $AB \geq 0$

$= 148$ km (nearest km) (1)

c) $\angle OBC = 180° - 118°$ (co-interior angles on parallel lines

$= 62°$. are supplementary)

$\tan \angle ABO = \dfrac{96}{112}$

$\angle ABO = 40.601$ (3dp.)

$\therefore \angle ABC = \angle OBC - \angle ABO$

$= 62° - 40.601°$

$= 21.399°\ldots$ (1)

$= 21°$

$\therefore$ bearing is $360° - 21°$

$= 339°T$ (1)

Section D.

1. a) degree: 3 1

leading coefficient: 1

b) $P(x) \times A(x) = (x^3+2x^2-5x-6)(x+2)$

$= x^4+2x^3+2x^3+4x^2-5x^2-10x-6x-12$

$= x^4+4x^3-x^2-16x-12$ 1

c)

$$\begin{array}{r|l} & x^2 \qquad -5 \\ \hline x+2 & x^3+2x^2-5x-6 \\ & \underline{x^3+2x^2} \\ & \qquad -5x-6 \\ & \qquad \underline{-5x-10} \\ & \qquad\qquad 4 \end{array}$$ 1

$\therefore P(x) \div A(x) = (x+2)(x^2-5)+4$ 1

d) test $x=1$, $x=-1$

$P(1) = 1^3+2(1)^2-5(1)-6$

$\neq 0$

$P(-1) = (-1)^3+2(-1)^2-5(-1)-6$

$= 0$ 1

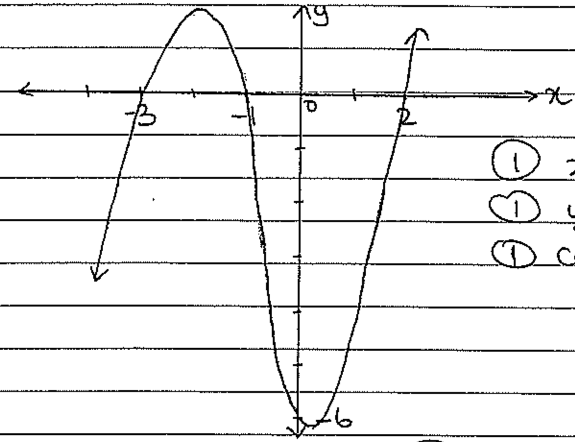
$\therefore x+1$ is a factor

$$\begin{array}{r|l} & x^2+x-6 \\ \hline x+1 & x^3+2x^2-5x-6 \\ & \underline{x^3+x^2} \\ & \quad x^2-5x \\ & \quad \underline{x^2+x} \\ & \qquad -6x-6 \\ & \qquad \underline{-6x-6} \\ & \qquad\quad 0 \end{array}$$ 1

$\therefore P(x) = (x+1)(x^2+x-6)$

$= (x+1)(x+3)(x-2)$ 1

e)



1 x-intercepts

1 y-intercept

1 correct shape & scale.

f) $-3 < x < -1$ OR $x > 2$ 1

2. a) $h = -16t^2+64t+80$

when $h=0$ 1

$16t^2-64t-80=0$

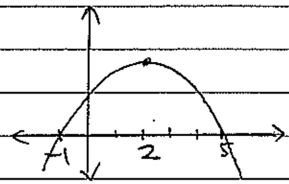
$t^2-4t-5=0$

$(t-5)(t+1)=0$

$t=5$ or -1 but $t \geq 0$

$\therefore$ it takes 5 seconds for the cannon ball to hit the water 1

b)



max. at $t = \frac{5-1}{2}$

$= 2$

when $t=2$

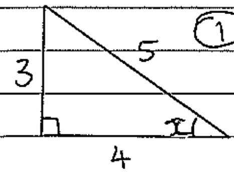
$h = -16(2)^2+64(2)+80$

$\therefore h = 144$ m.

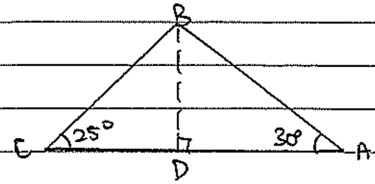
Section E

1. let $P(x) = x^3 + 2x^2 - 5ax + 7$
   and $Q(x) = x^3 + ax^2 - 12x + 6$
   $P(-1) = (-1)^3 + 2(-1)^2 - 5a(-1) + 7$
   $= 5a - 6$
   $P(-1) = A$
   $\therefore A = 5a - 6$ (1)
   $Q(2) = 2^3 + a \times 2^2 - 12 \times 2 + 6$
   $= 4a - 10$
   $Q(2) = B$
   $\therefore B = 4a - 10$ (1)
   Since $2A + B = 6$
   $2(5a-6) + (4a-10) = 6$
   $10a - 12 + 4a - 10 = 6$
   $14a = 28$
   $\therefore a = 2$ (1)

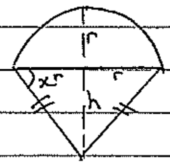
2. ABCD is a rectangle.
   $\angle DAE = \angle ADF = 90^\circ$ (angles in a rectangle are right angles)
   In ΔADF & ΔDAE
   $\angle ADF = \angle DAE = 90^\circ$ (proven above)
   AD is common (1)
   AF = DE (given)
   $\therefore \Delta ADF \equiv \Delta DAE$ (RHS)
   $\therefore$ AE = DF (corresponding sides of congruent triangles are equal) (1)
   $\angle DAE + \angle ADF = 90^\circ + 90^\circ$
   $= 180^\circ$
   $\therefore$ AE // DF (co-interior angles are supplementary) (1)
   $\therefore$ AEFD is a rectangle(one pair of opposite sides, AE & DF are equal and parallel and the diagonals, AF & DE are equal) (1)

3. 

(1)
$\sin x = \frac{3}{5}$ (1)
$\cos x = \frac{4}{5}$ (1)

4. 

$\sin 30 = \frac{BD}{28}$
$BD = 28 \sin 30$
$\sin 25 = \frac{BD}{CB}$ (1)
$\therefore CB = \frac{BD}{\sin 25}$
$= \frac{28 \sin 30}{\sin 25}$
$= 33.1268\ldots$ (1)
$= 33.13$ m (2dp.)

5. 

let the radius of the circle be r
let the perpendicular height of the triangle be h. (1)
Area of semi-circle $= \frac{1}{2}\pi r^2$
Area of triangle $= \frac{1}{2} \times h \times 2r$
$= hr$
Areas are equal,
$\therefore \frac{1}{2}\pi r^2 = hr$ (1)
$h = \frac{1}{2}\pi r$
$\tan x = \frac{h}{r}$
$= \frac{\frac{1}{2}\pi r}{r}$
$\therefore \tan x = \frac{1}{2}\pi$ (1)