| Name: |  |
| :--- | :--- |
| Class: |  |

## YEARLY EXAMINATION

## YEAR 92017

## MATHEMATICS

Time Allowed - 100 minutes plus 5 minutes Reading time.

## INSTRUCTIONS:

- Start each section on a new page
- Write your Name and Class at the top of each page
- Write in Pen and draw diagrams in Pencil
- Department of Education approved calculators are permitted
- The use of mathematical templates are permitted.
- Show all necessary working
- Marks may not be awarded for untidy or carelessly arranged work
- No grid paper is to be used unless provided with the examination paper
- Teachers: Please collect each section separately.

| Outcome | MC | A |  | B |  | C |  | D |  | E |  | Total |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Measurement |  |  |  |  |  | $1-3$ | $/ 8$ | $1-2$ | $/ 7$ | 4 | $/ 4$ | $/ 19$ |
| Number |  | 1 | $/ 2$ |  |  | $4-5$ | $/ 6$ |  |  |  |  | $/ 8$ |
| Algebra |  | $2-3$ | $/ 6$ | 1 | $/ 8$ |  |  | $3-4$ | $/ 10$ | 1 | $/ 3$ | $/ 27$ |
| Coord. Geom |  |  |  | 2 | $/ 9$ |  |  |  |  | 2 | $/ 4$ | $/ 13$ |
| Geometry |  | 4 | $/ 4$ |  |  |  |  |  |  | 3 | $/ 6$ | $/ 10$ |
| Stats and Prob |  | 5 | $/ 5$ |  |  | 6 | $/ 3$ |  |  |  |  | $/ 8$ |
| MC | $/ 5$ |  |  |  |  |  |  |  |  |  |  | $/ 5$ |
| Total | $/ 5$ |  | $/ 17$ |  | $/ 17$ |  | $/ 17$ |  | $/ 17$ |  | $/ 17$ | $/ 90$ |

## Multiple Choice - Answer on the Multiple-Choice Answer Sheet Provided

## Question 1

Which of the following is a sufficiency proof for a rhombus?
a) Diagonals are perpendicular to each other
b) All sides are equal
c) Opposite sides are parallel
d) Diagonals bisect each other

## Question 2

For a set of normally distributed data of 300 scores, how many scores are expected to have a z-score between -2 and 1 ?
a) 285
b) 245
c) 204
d) 180

## Question 3

What is the value of $\theta^{\circ}$ ?

NOT TO SCALE

a) $96^{\circ}$
b) $102^{\circ}$
c) $106^{\circ}$
d) $112^{\circ}$

## Question 4

How much interest is earned when $\$ 1500$ is invested at $4 \%$ p.a. compounded monthly for 30 years?
a) $\$ 4970$
b) $\$ 3470$
c) $\$ 4865$
d) $\$ 3365$

## Question 5

Which if the following is the graph of $y=2(x+1)(x-3)^{2}(2 x-1)$ ?
a)

b)

c)

d)


## Section A: (17 Marks) Answer on your own writing paper

1. Rewrite $\frac{3+\sqrt{2}}{1+\sqrt{2}}$ in the form of $a+\sqrt{b}$ where $a$ and $b$ are both integers.
2. Expand and simplify:
a) $(3 x-4 y)(2 x+y)$
b) $(2 a+b)(b-4)-(b+3)(a-b-1)$
3. Factorise fully:
a) $8 x^{3}+y^{3}$
b) $4 a x-5 y b-4 b x+5 a y$
4. In the diagram below, $A$ and $B$ are points on the circle with centre $O . M$ is the midpoint of $A B$

a) Prove that $\triangle A O M \equiv \triangle B O M$ using SSS.
b) Hence, prove that the line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.
5. A group of 6 students took a test and got the following results:

$$
4,5,7,8,8,10
$$

a) What is the mean of the scores?
b) What is the standard deviation of the scores?
c) What is the $z$-score of the lowest performing student in the group?
d) A new student joins the class and took the same test. His score is 3 marks higher than the new mean. What score did the new student get?

## Section B: (17 Marks) Start a new sheet of paper

1. Solve for the pronumerals of the following:
a) $5 x-7=1-3 x \quad 2$
b) $\sqrt{x+11}=1-x$
c) $3^{2 x-y}=27$
$2^{3 x+2 y}=4^{-3}$
2. a) On a number plane, clearly label the points $A(2,1)$ and $B(6,-1)$. (You may add to this diagram as the question progresses if necessary).
b) Show that $M$, the midpoint of $A B$, has coordinates ( 4,0 ).
c) Show that $l_{1}$, the perpendicular bisector of AB is given by $2 x-y-8=0$.
d) Show that $\mathrm{C}(5,2)$ lies on $l_{1}$.
e) Is $\triangle A B C$ an equilateral, isosceles or scalene triangle? Justify your answer.
f) Find the coordinates of $D$, such that $A B C D$ is a parallelogram. Show all necessary working.

## Section C: (17 Marks) Start a new sheet of paper

1. Find the exact value of:
a) $\sin 45^{\circ}$
b) $\cos 210^{\circ}$
2. At 11am, a ship begins to set sail on a bearing of 130T from a position that is 50 km due west of a lighthouse. The ship travels at a speed of $10 \mathrm{~km} / \mathrm{h}$. At what time will the ship be directly South of the lighthouse? (Answer to the nearest minute).
3. a) What is the surface area of a closed cone with base radius 9 cm and perpendicular height 40 cm ?
b) If the dimensions of the cone are increased by $50 \%$, what percentage will the surface area have increased by?
4. A car was bought for $\$ 25000$. Each year its value depreciates by $12 \%$.

What is the market value of the car after 10 years? Express your answer correct to 5 significant figures.
5. Nick works as a real estate agent and receives a commission of $1 \%$ of all the sales he makes. In the financial year 2016-2017 he sold a total of $\$ 8.5$ million dollars worth of homes. In addition, Nick also earned \$1600 worth of interest from the bank, while spending $\$ 800$ on work expenses and making $\$ 500$ worth of donations that are tax deductible.
a) Calculate Nick's taxable income for the financial year 2016-2017.
b) Use the table below to work out the tax payable for Nick for the year.
Resident tax rates 2017-18

| Taxable income | Tax on this Income |
| :--- | :--- |
| $0-\$ 18,200$ | Nil |
| $\$ 18,201-\$ 37,000$ | 19c for each $\$ 1$ over $\$ 18,200$ |
| $\$ 37,001-\$ 87,000$ | $\$ 3,572$ plus 32.5 c for each $\$ 1$ over $\$ 37,000$ |
| $\$ 87,001-\$ 180,000$ | $\$ 19,822$ plus 37 c for each $\$ 1$ over $\$ 87,000$ |
| $\$ 180,001$ and over | $\$ 54,232$ plus $\mathbf{4 5 c}$ for each $\$ 1$ over $\$ 180,000$ |

6. Helen bought 2 raffle tickets for a fundraiser. 3 tickets are drawn out of 100 for 3
different prizes. By drawing a probability tree diagram, find the probability of Helen winning exactly 1 prize?

## Section D: (17 Marks) Start a new sheet of paper

1. The angle of depression from the top of a tower to Carmen's feet is $55^{\circ}$. Carmen then walks a further 50 m away from the tower, and from there the angle of elevation to the top of the tower is $35^{\circ}$.
a) Draw a neat diagram to illustrate all the information given above.
b) Find the height of the tower correct to 1 decimal place.
c) Using the 1 decimal place answer in part b, find to the nearest metre, the original distance between Carmen and the base of the tower.
2. Sketch the graph of $y=\tan x$ for $0^{\circ} \leq x \leq 360^{\circ}$
3. A window frame consisting of 6 congruent rectangles is illustrated below. Only 12 metres of frame is available for its construction.

a) Show that the area of the window is given by $A=3 w-\frac{3 w^{2}}{4}$ where $w$ is the width of the window.
b) Draw the graph $A=3 w-\frac{3 w^{2}}{4}$ showing all important features
c) Hence or otherwise, find the maximum area of the window.
4. a) By using the factor theorem and long division, factorise the function to linear factors:

$$
f(x)=4 x^{3}-8 x^{2}-x+2
$$

b) Find the remainder when $f(x)$ is divided by $4 x-3$

## Section E: (17 Marks) Start a new sheet of paper

1. When a polynomial is divided by $x-p$, the remainder is $p^{2}$. When the same polynomial

3 is divided by $x-q$, the remainder is $q^{2}$. What is the remainder when the polynomial is divided by $x^{2}-(p+q) x+p q$ ? $(p \neq q)$.
2. a) Without finding the points of intersection, sketch and shade the region bounded by (and inclusive of) the following lines:

$$
\begin{gathered}
y=-\frac{1}{2} \\
x-y=10 \\
x=4-\frac{y}{2}
\end{gathered}
$$

b) Write the correct set of inequalities to represent the region.
3. Two squares $A B C D$ and $A E F G$ are drawn above. $A G$ and $E B$ intersects at $K$ and $D G$ and $A B$ intersect at H . Let $\angle A D G=\alpha$.


Not to scale
a) Copy the diagram neatly onto your paper and prove that $\triangle A D G \equiv \triangle A B E$
b) Prove that $E B \perp D G$.
4.

$\triangle A B C$ is a scalene triangle with $A E \perp B C . A D$ is the median from $A$ such that $B D=D C$.
a) Show that $A D^{2}-D E^{2}=A B^{2}-B E^{2} \quad 1$
b) Hence or otherwise, Prove that $A B^{2}+A C^{2}=2\left(A D^{2}+B D^{2}\right)$ 3

